Online Appendix for "Gambler's Fallacy and Imperfect Best Response in Legislative Bargaining"

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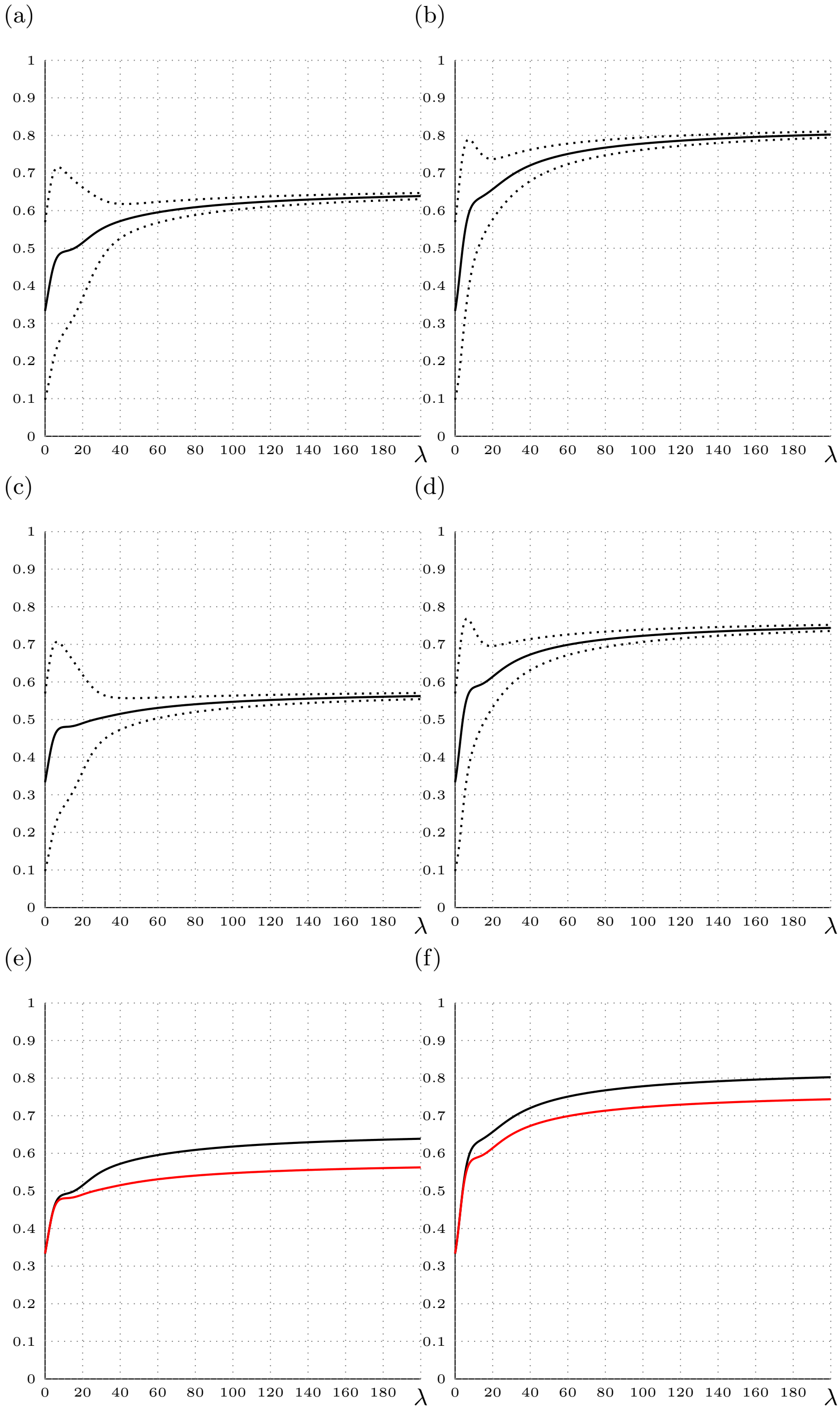
Contents

This document contains supplementary material for 'Gambler's Fallacy and Imperfect Best Response in Legislative Bargaining'.

1 Numerical Computations of QRE and QGF: Additional Results

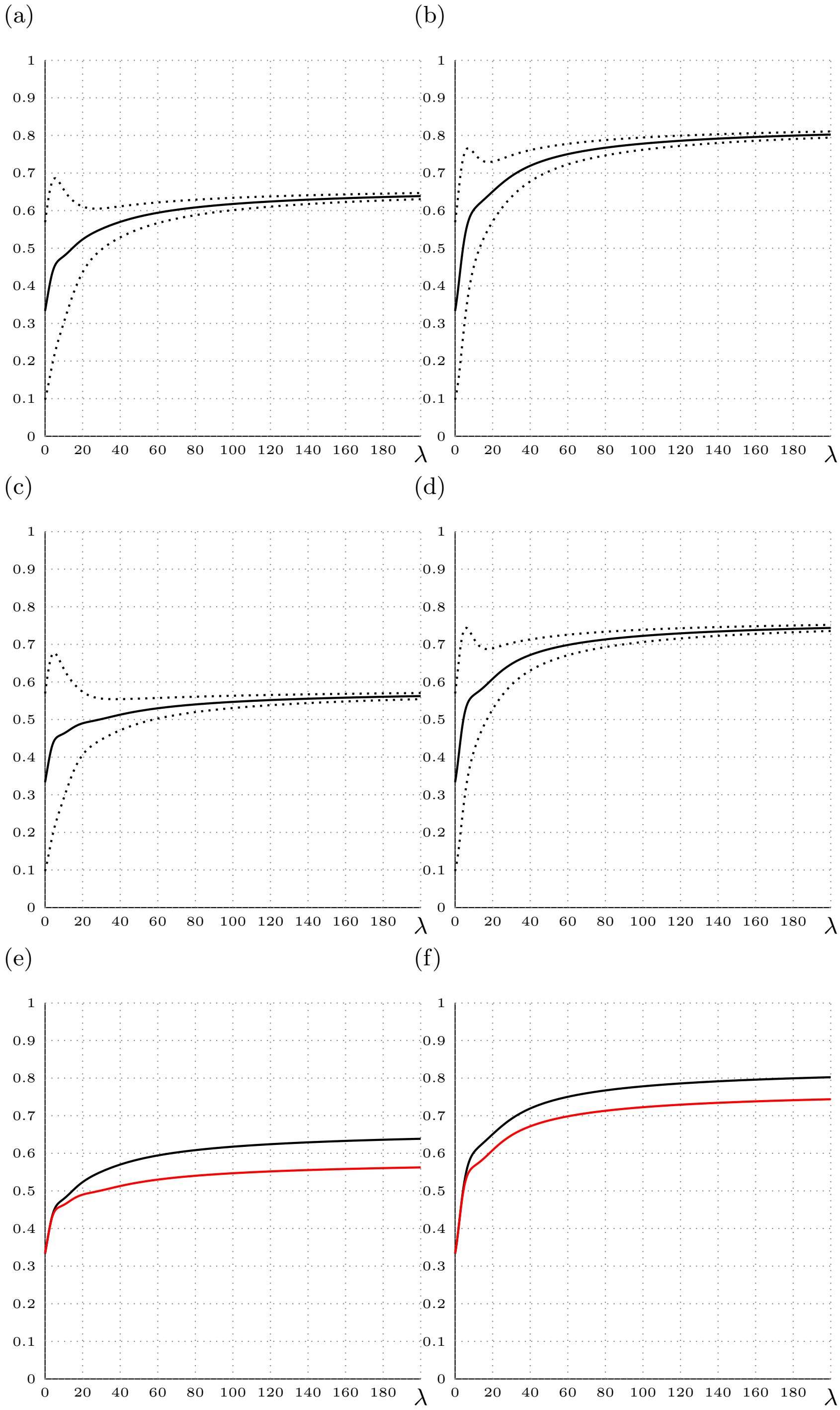
All figures with $n = 3$ and $\phi = 1$ (for QGF). Unless otherwise specified, the first row of the following figures (panels (a), (b)) shows the QRE, the second row (panels (c), (d)) shows the QGF and the third row (panels (e), (f)) shows both, for comparison. The first column of the figures (panels (a), (c), (e)) shows results for $\delta = 1$ and the second column (panels (b), (d), (f)) shows results for $\delta = \frac{1}{2}$.

Figure 1: QRE vs. QGF in Baron and Ferejohn (1989) model
Mean proposer's share



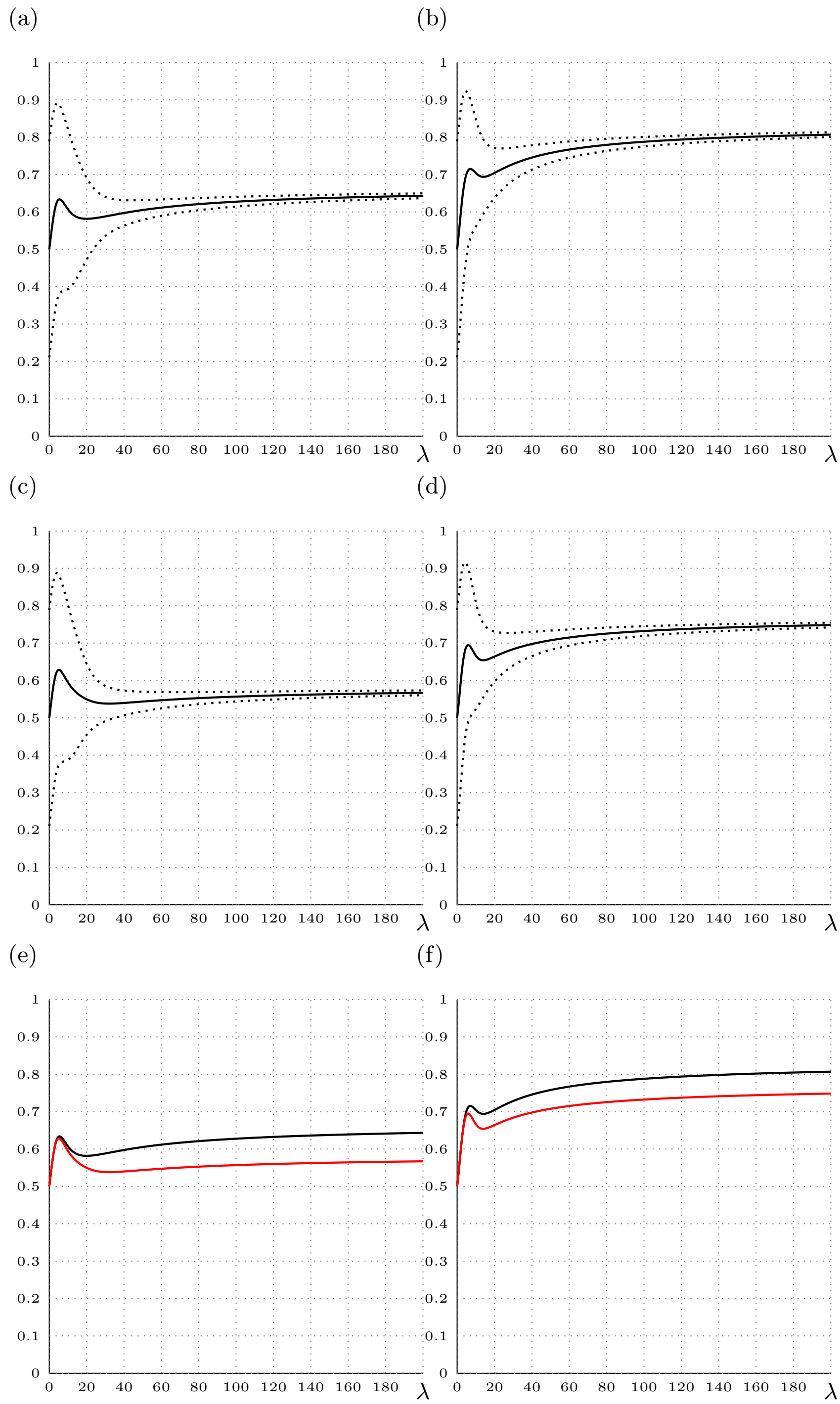
Note: Mean share proposer allocates to herself (solid lines) ± one standard deviation (dotted lines). Based on proposed allocations.

Figure 2: QRE vs. QGF in Baron and Ferejohn (1989) model
Mean approved proposer's share



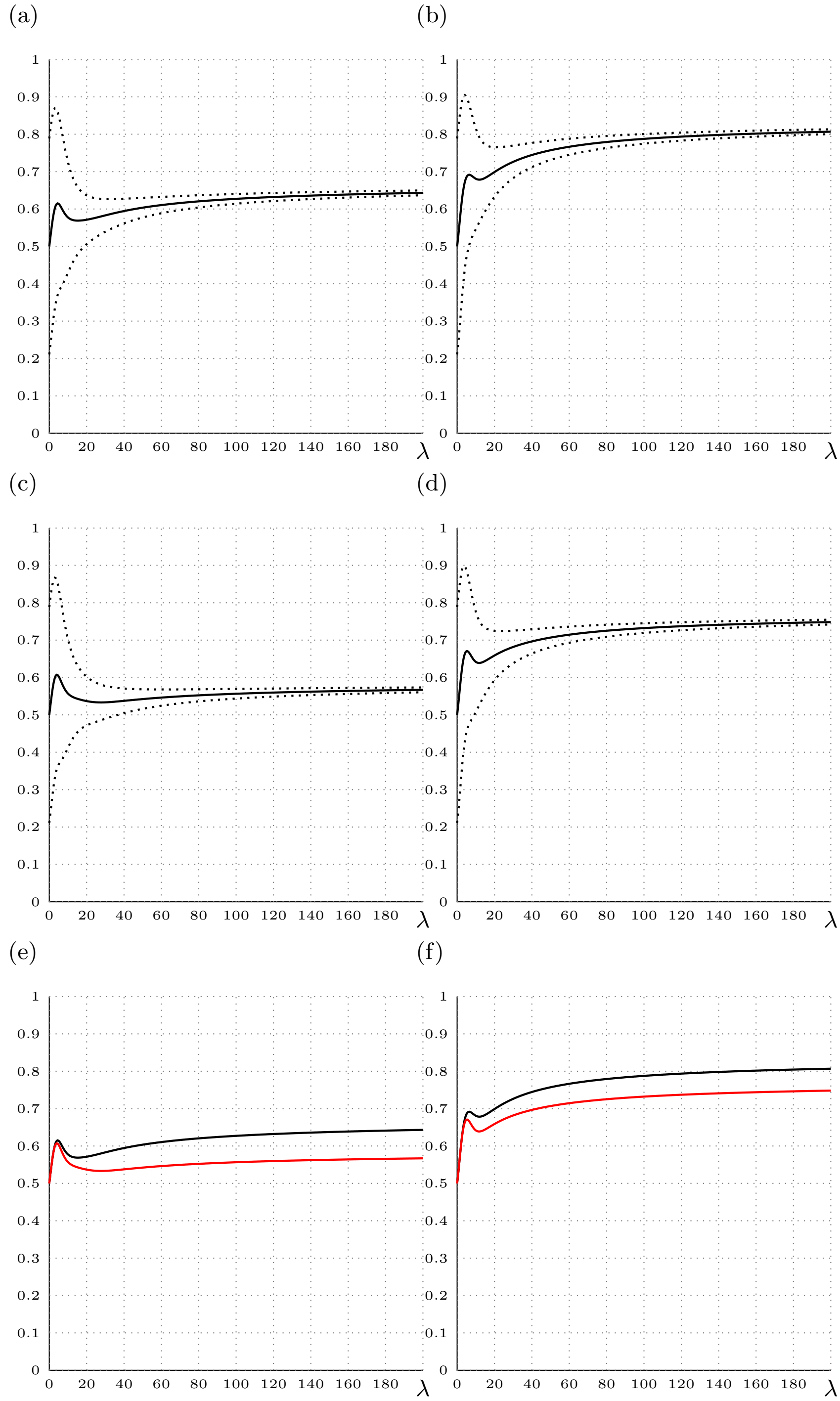
Note: Mean share proposer allocates to herself (solid lines) $\pm$ one standard deviation (dotted lines). Based on proposed allocations conditional on acceptance.

Figure 3: QRE vs. QGF in Baron and Ferejohn (1989) model
Mean proposer's share conditional on MWC



Note: Mean share proposer allocates to herself (solid lines) ± one standard deviation (dotted lines). Based on proposed allocations conditional on approximate (no more than 5%) minimum winning coalition.

Figure 4: QRE vs. QGF in Baron and Ferejohn (1989) model
Mean approved proposer's share conditional on MWC



Note: Mean share proposer allocates to herself (solid lines) ± one standard deviation (dotted lines). Based on proposed allocations conditional on acceptance and on approximate (no more than 5%) minimum winning coalition.

Figure 5: QRE vs. QGF in Baron and Ferejohn (1989) model
Equilibrium continuation value v^*

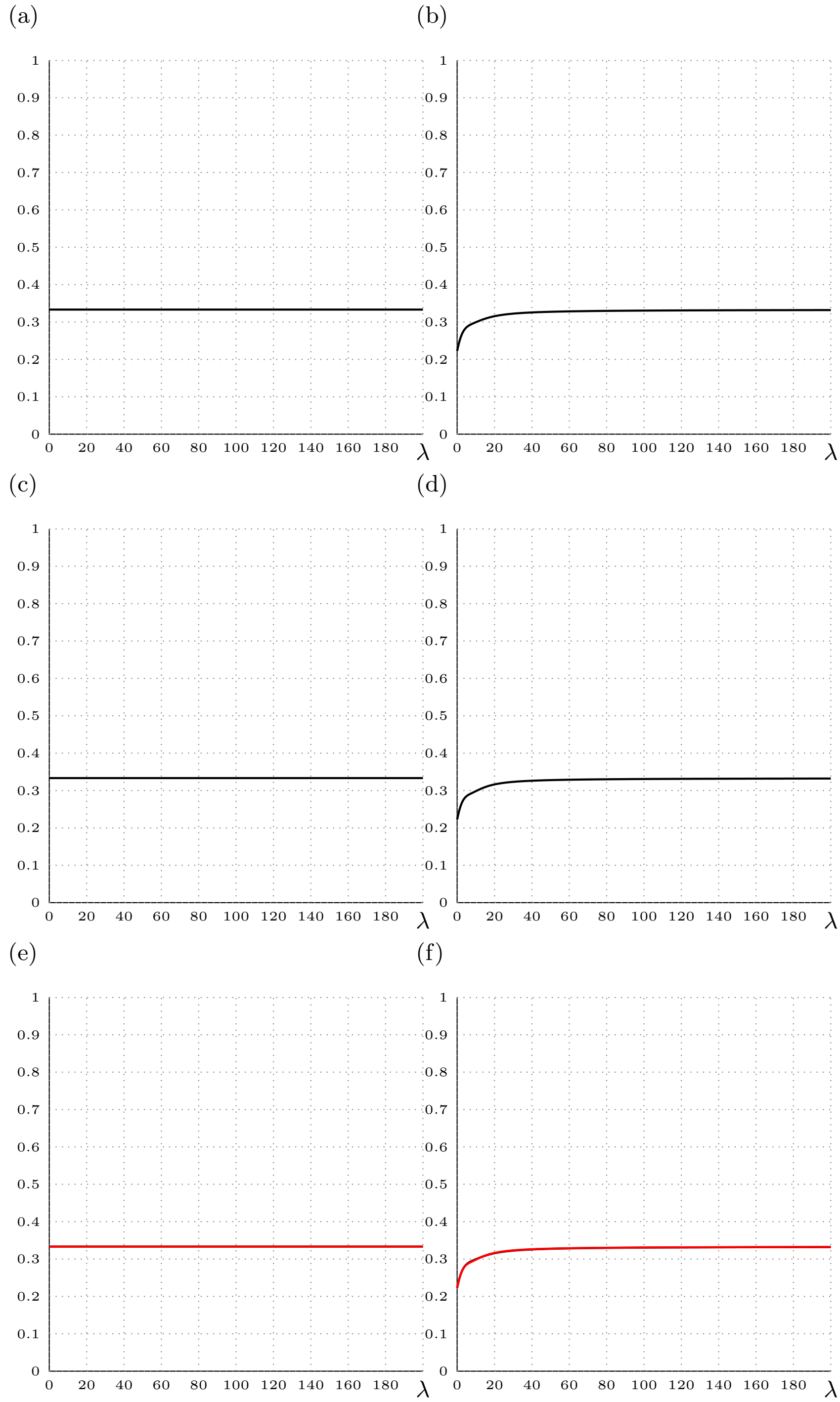
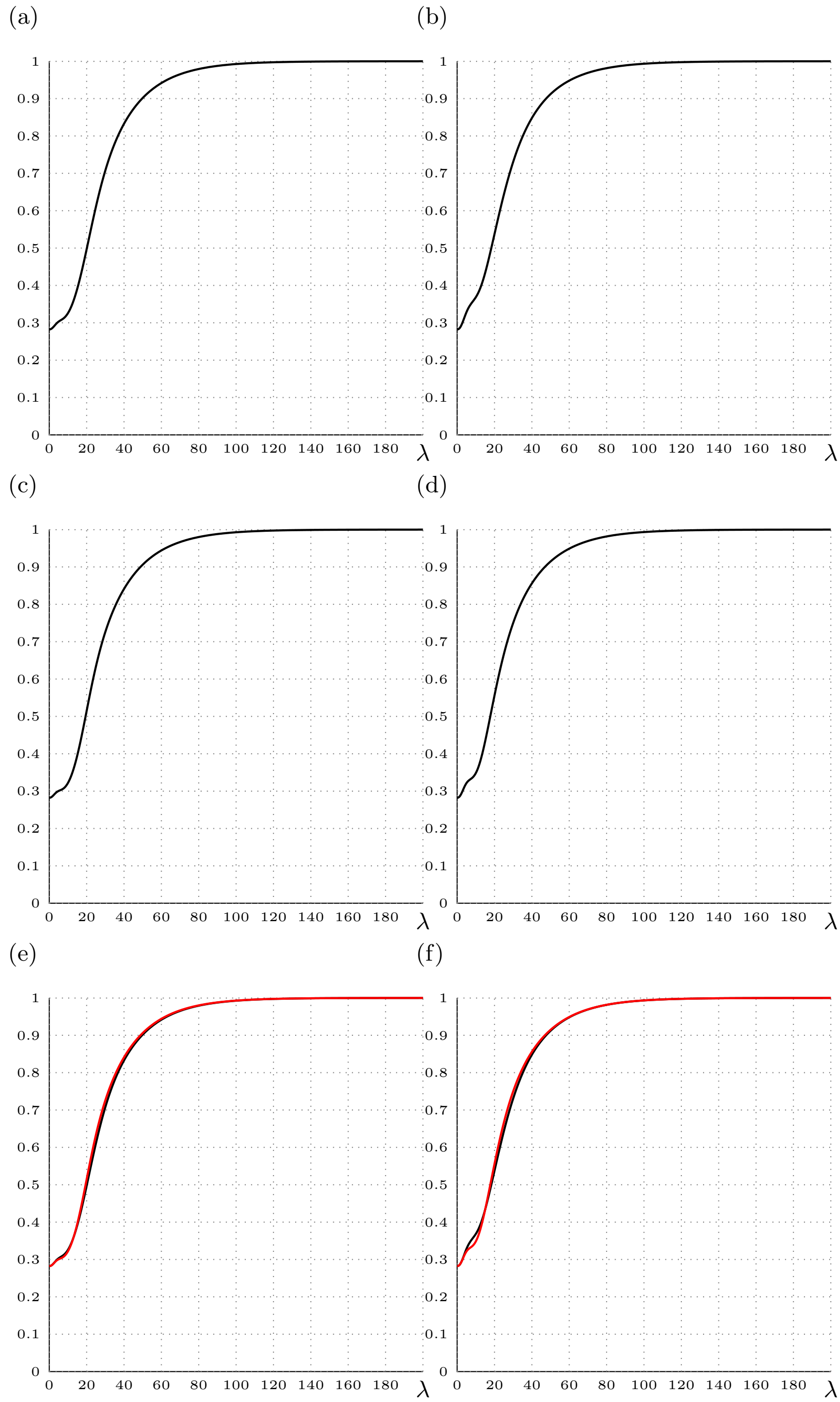
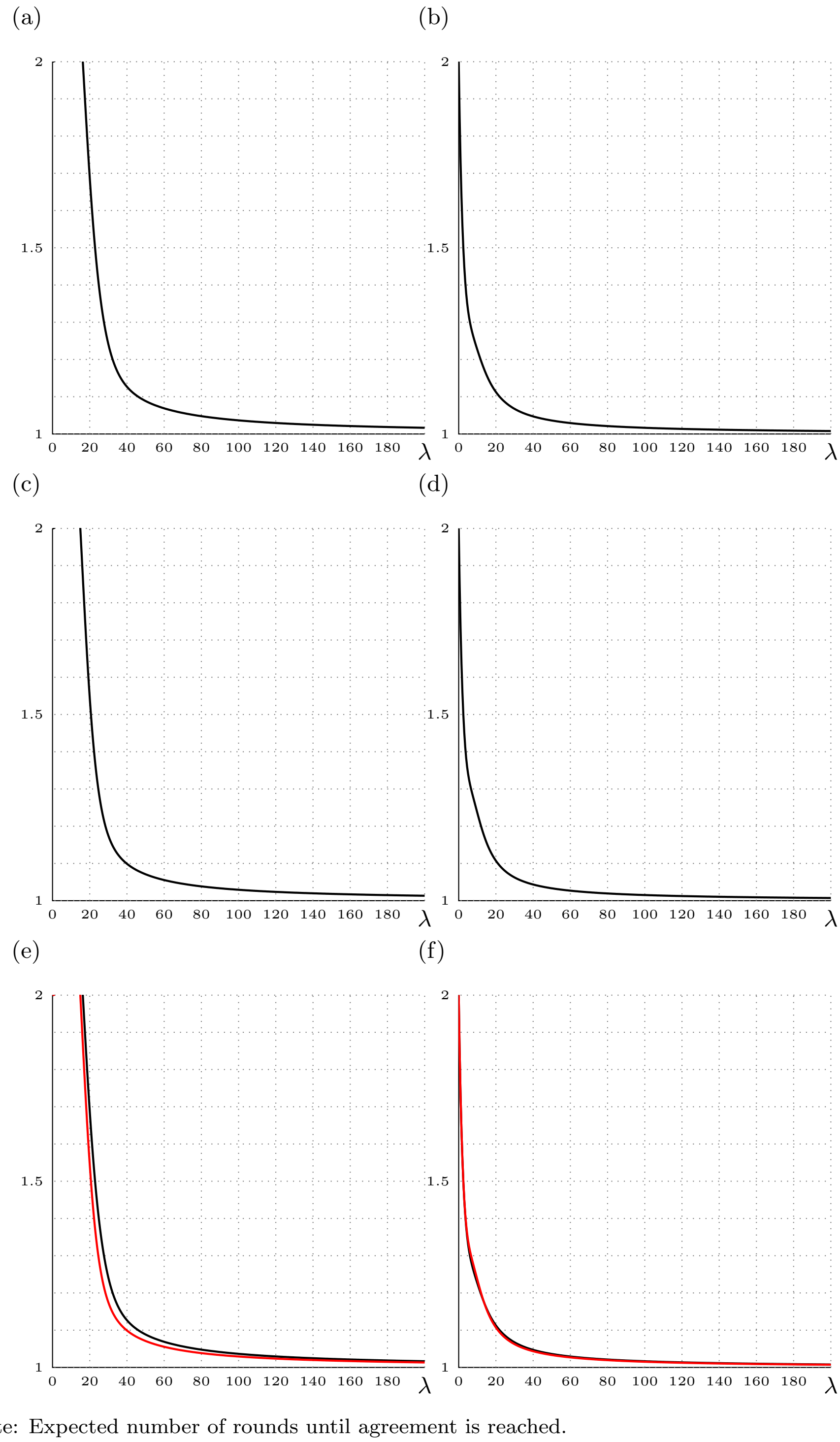


Figure 6: QRE vs. QGF in Baron and Ferejohn (1989) model
MWC frequency



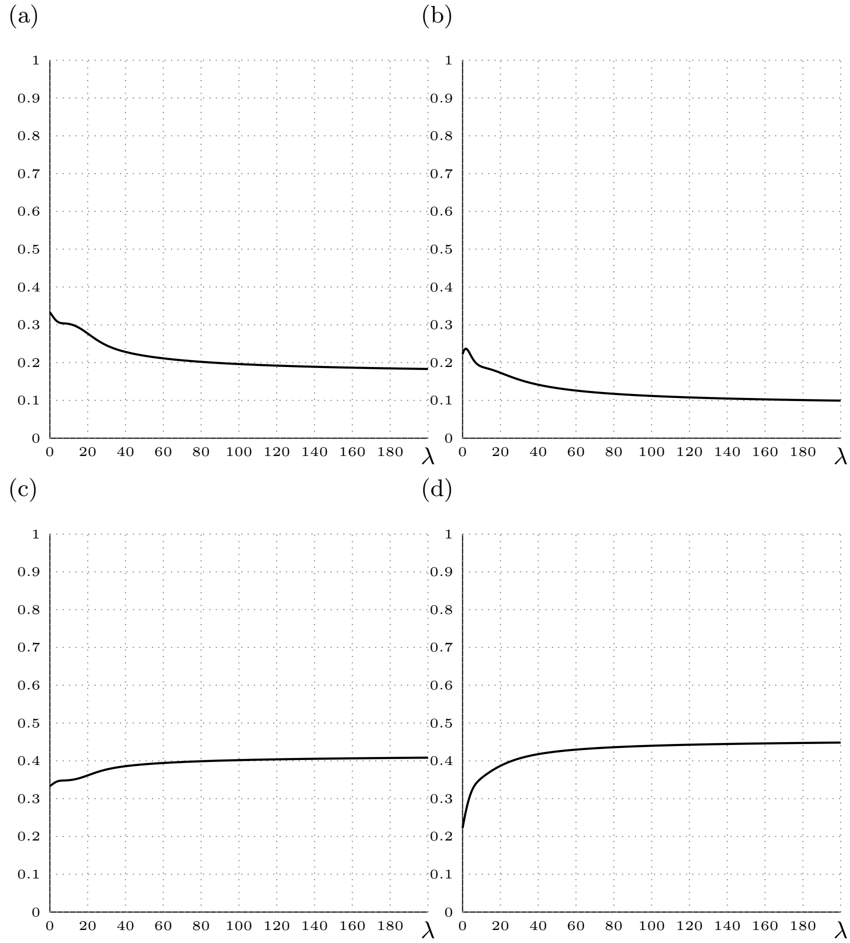
Note: Frequency of approximate (no more than 5%) minimum winning coalitions.

Figure 7: QRE vs. QGF in Baron and Ferejohn (1989) model
Expected number of rounds



Note: Expected number of rounds until agreement is reached.

Figure 8: QRE vs. QGF in Baron and Ferejohn (1989) model
Equilibrium continuation value in QGF

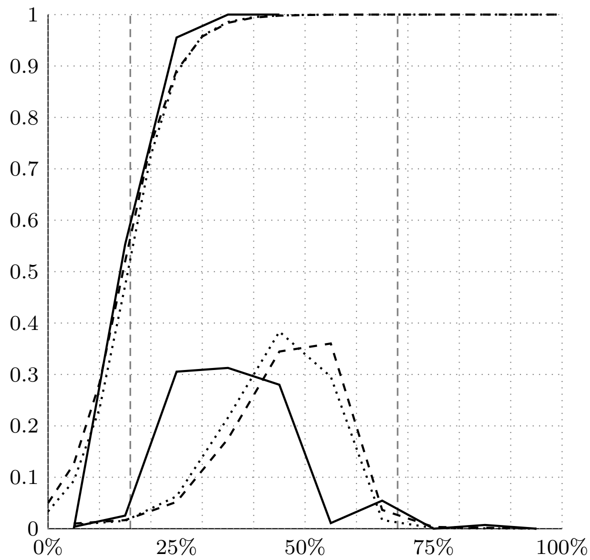


Note: Equilibrium odd round v^* in QGF. First row v^* of proposing player. Second row v^* of non-proposing player. Left column $\delta = 1$. Right column $\delta = \frac{1}{2}$.

2 Empirical vs. Estimated Distributions

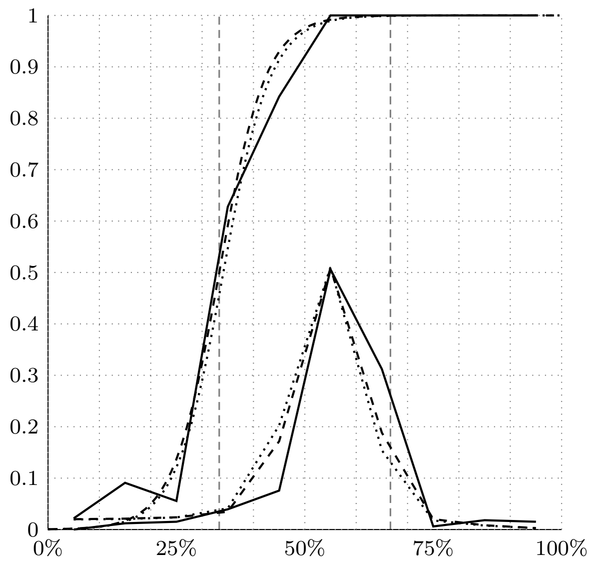
Each figure of this section shows the distribution of the proposer's share and the probability of approving the proposed allocation derived from the experimental data (round 1 observations only) and predicted by QRE and QGF at the benchmark MLE $\hat{\lambda}$ and $\{\hat{\lambda}, \hat{\phi}\}$ respectively.

Figure 9: Distribution of proposer's share and probability of approving vote
Experimental data vs. QRE vs. QGF



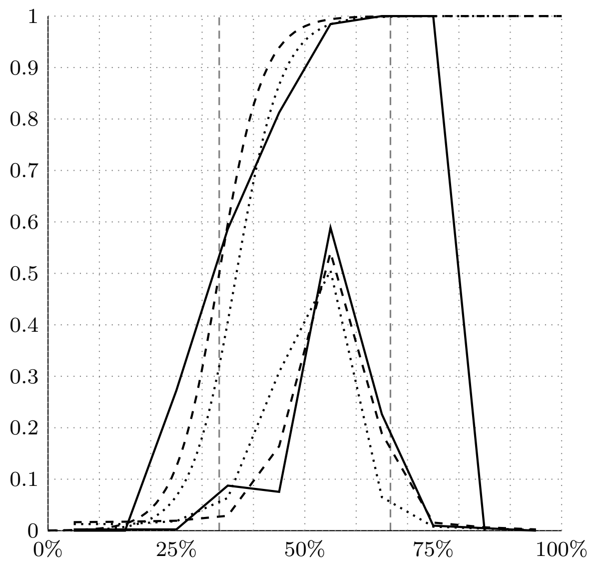
Note: Experiment 1 (Frechette, Kagel, and Lehrer, 2003). Dashed line QRE with $\hat{\lambda} = 20.2$. Dotted line QGF with $\hat{\lambda} = 21.7$ and $\hat{\phi} = 1$. Vertical lines benchmark SSPE predictions (v^* and proposer's share).

Figure 10: Distribution of proposer's share and probability of approving vote
Experimental data vs. QRE vs. QGF



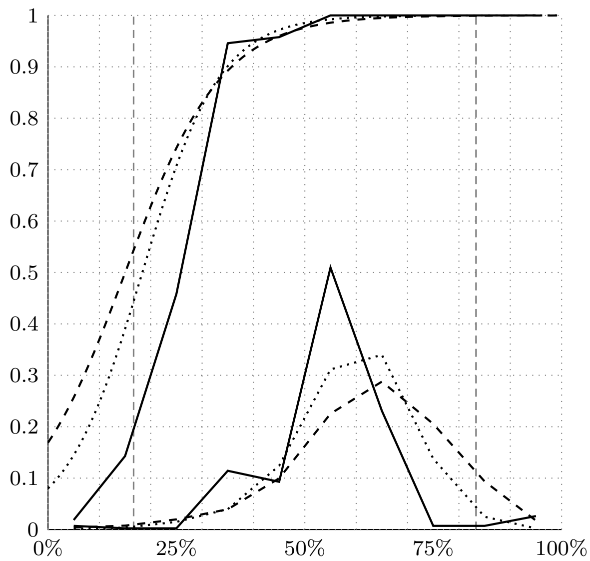
Note: Experiment 2 (Frechette, Kagel, and Morelli, 2005b). Dashed line QRE with $\hat{\lambda} = 22.1$. Dotted line QGF with $\hat{\lambda} = 21.7$ and $\hat{\phi} = 3$. Vertical lines benchmark SSPE predictions (v^* and proposer's share).

Figure 11: Distribution of proposer's share and probability of approving vote
Experimental data vs. QRE vs. QGF



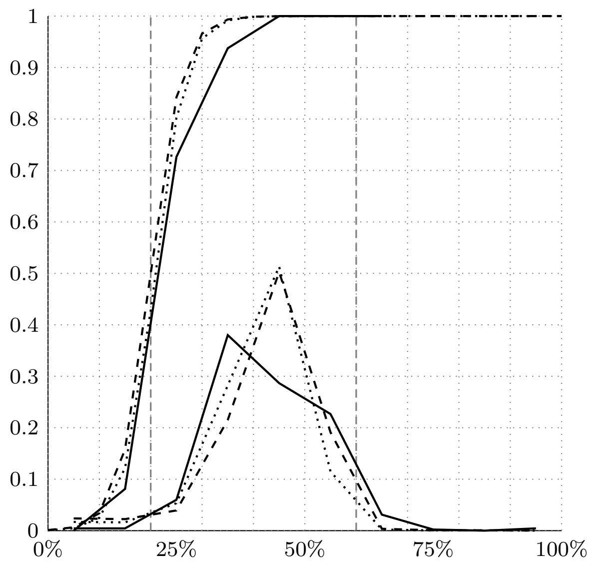
Note: Experiment 3 (Frechette, Kagel, and Morelli, 2005b). Dashed line QRE with $\hat{\lambda} = 23.4$. Dotted line QGF with $\hat{\lambda} = 22.5$ and $\hat{\phi} = 1$. Vertical lines benchmark SSPE predictions (v^* and proposer's share).

Figure 12: Distribution of proposer's share and probability of approving vote
Experimental data vs. QRE vs. QGF



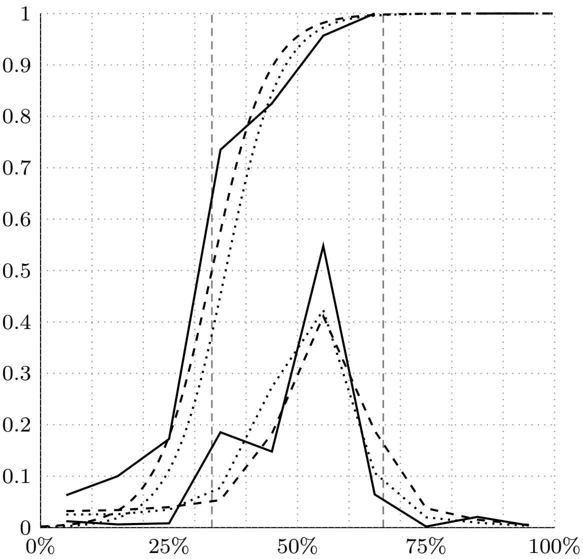
Note: Experiment 4 (Frechette, Kagel, and Morelli, 2005b). Dashed line QRE with $\hat{\lambda} = 10.6$. Dotted line QGF with $\hat{\lambda} = 13.3$ and $\hat{\phi} = 1$. Vertical lines benchmark SSPE predictions (v^* and proposer's share).

Figure 13: Distribution of proposer's share and probability of approving vote
Experimental data vs. QRE vs. QGF



Note: Experiment 5 (Frechette, Kagel, and Morelli, 2005a). Dashed line QRE with $\hat{\lambda} = 33.5$. Dotted line QGF with $\hat{\lambda} = 33.8$ and $\hat{\phi} = 1$. Vertical lines benchmark SSPE predictions (v^* and proposer's share).

Figure 14: Distribution of proposer's share and probability of approving vote
Experimental data vs. QRE vs. QGF

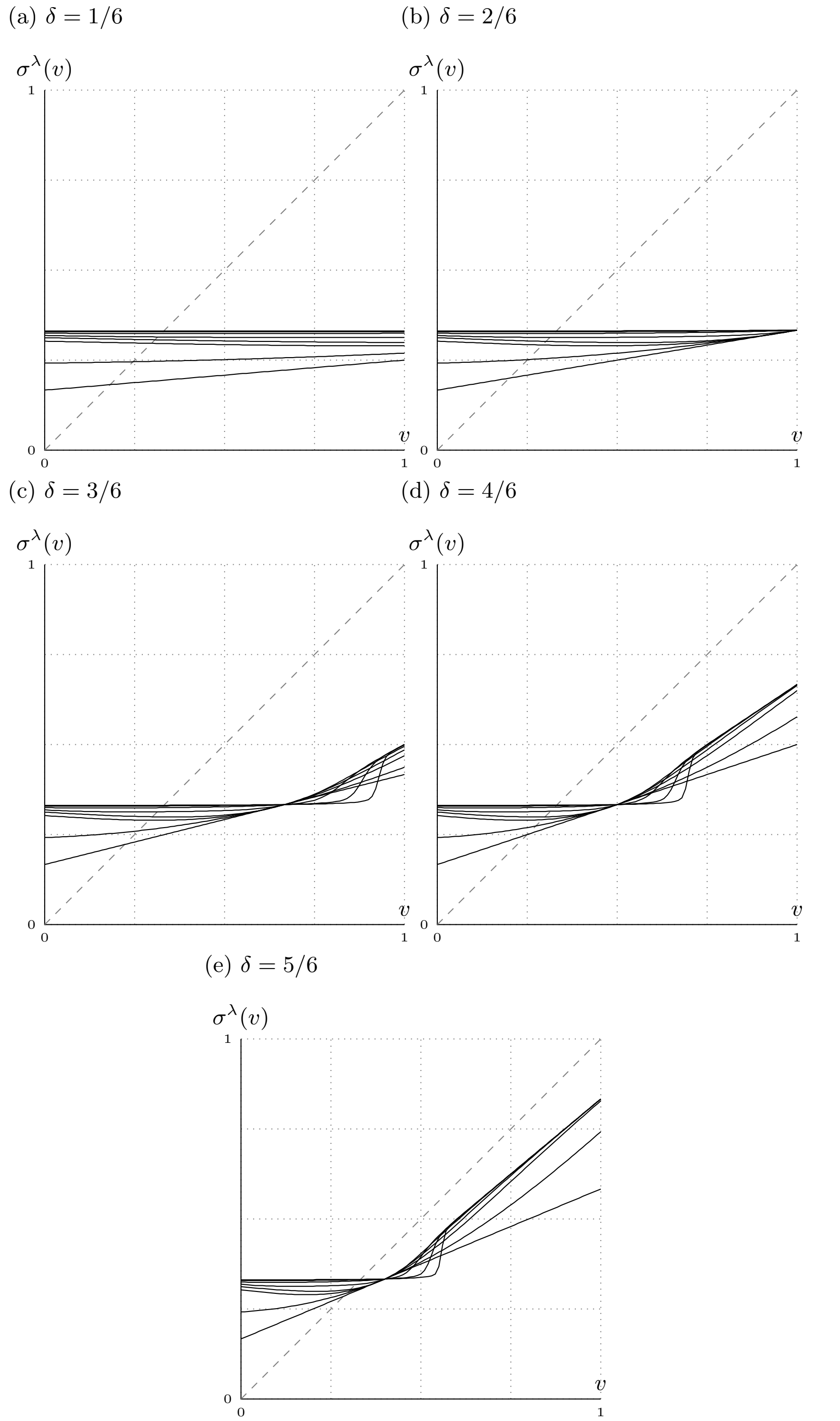


Note: Experiment 6 (Drouvelis, Montero, and Sefton, 2010). Dashed line QRE with $\hat{\lambda} = 18.4$. Dotted line QGF with $\hat{\lambda} = 18.8$ and $\hat{\phi} = 1$. Vertical lines benchmark SSPE predictions (v^* and proposer's share).

3 Systematic Search for QRE Multiplicity

The figures show the systematic search for multiplicity of QRE. Each figure plots $\sigma^\lambda(v)$ for $v \in [0, 1]$ and $\lambda \in \{0, 2, 6, 10, 18, 36, 72, 144\}$ for the model with $n = 3$.

Figure 15: QRE $\sigma(v)$ mapping



Note: $\sigma^\lambda(v)$ mapping for $\lambda \in \{0, 2, 6, 10, 18, 36, 72, 144\}$ (more responsive for larger λ) and $v \in [0, 1]$. Dashed line is 45° degree line.

4 Optimal QRE Proposals

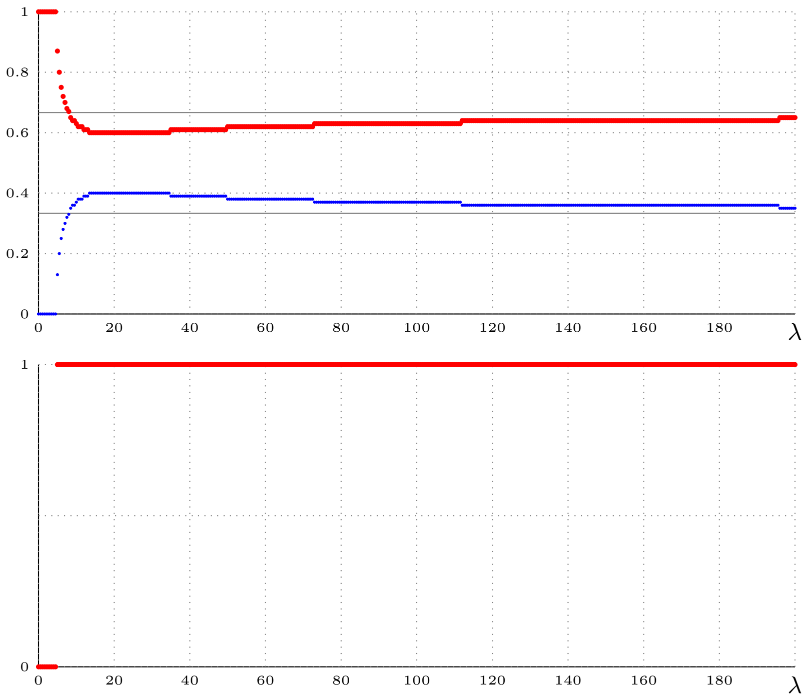
The following figures characterize optimal QRE proposals for $\lambda \in [0, 200]$ and all possible combinations of $n \in \{3, 5\}$ and $\delta \in \{\frac{1}{2}, 1\}$. Optimal proposal $x^* \in X'$ maximizes

$$x_i p_{v^*}^{\lambda}(x) + \delta v^*(1 - p_{v^*}^{\lambda}(x))$$

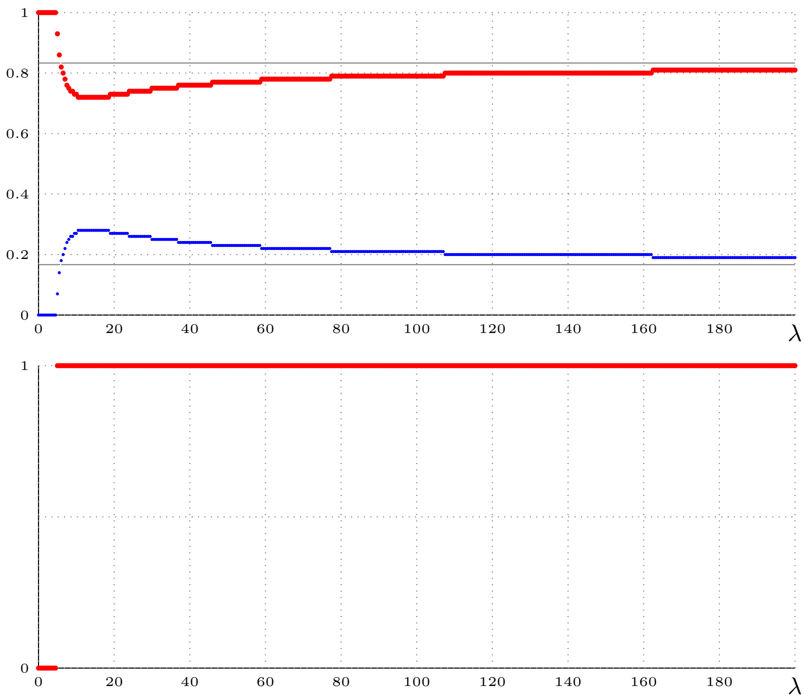
where v^* is the equilibrium continuation value and $p_{v^*}^{\lambda}(x)$ is the equilibrium probability of proposal x being accepted. x^* can be characterized by two numbers: x_i^* the proposer allocates to herself (top figure of each panel) and n^*, the number of coalition partners with strictly positive shares in x^* (bottom figure of each panel).

Figure 16: QRE in Baron and Ferejohn (1989) . . . Optimal proposal

(a) $n = 3$ & $\delta = 1$



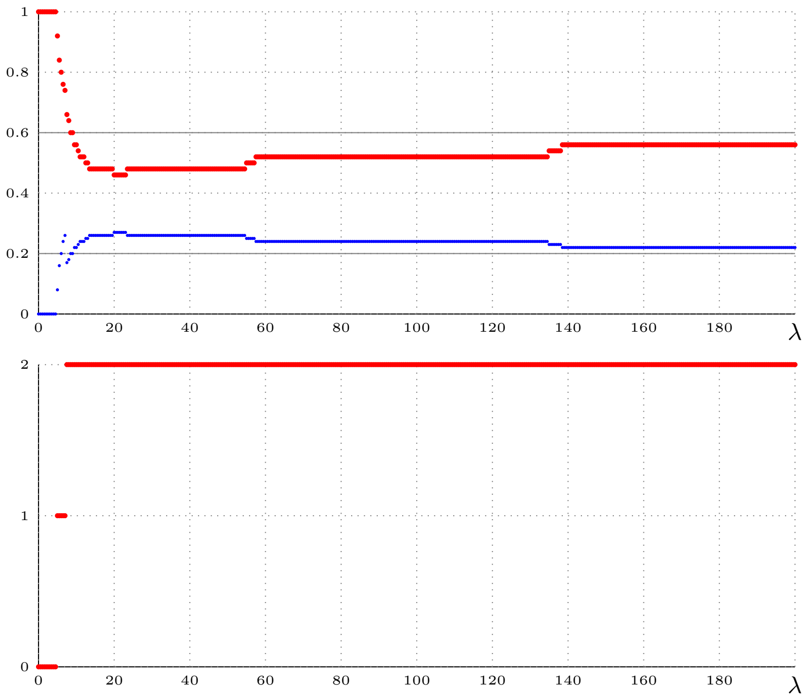
(b) $n = 3$ & $\delta = \frac{1}{2}$



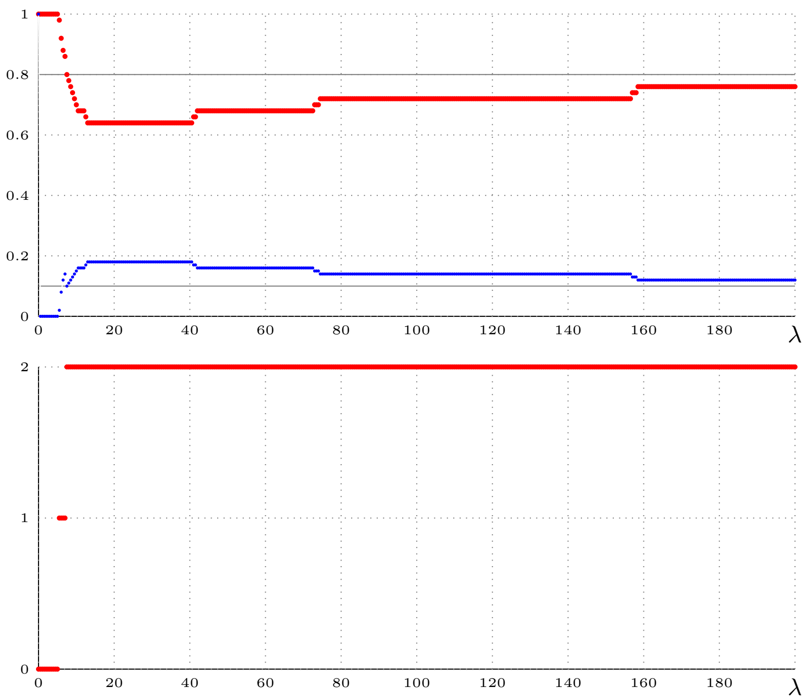
Note: Top figure of each panel is proposer's share (thick line) and responder's share (thin line) in proposer's optimal proposal. Bottom figure of each panel is # of coalition partners with strictly positive shares in optimal proposal.

Figure 17: QRE in Baron and Ferejohn (1989) . . . Optimal proposal

(a) $n = 5$ & $\delta = 1$



(b) $n = 5$ & $\delta = \frac{1}{2}$



Note: Top figure of each panel is proposer's share (thick line) and responder's share (thin line) in proposer's optimal proposal. Bottom figure of each panel is # of coalition partners with strictly positive shares in optimal proposal.

5 QRE Expected Payoffs

The following figures show the proposer's and responder's expected payoffs in a QRE for $n \in \{3, 5\}$ and $\delta \in \{\frac{1}{2}, 1\}$. Denote the proposer by i. The proposer's expected payoffs in a QRE with continuation value v^* is

$$\sum_{x \in X'} r^{\lambda}_{v^*,i}(x) \left[x_i p^{\lambda}_{v^*,i}(x) + \delta v^* (1 - p^{\lambda}_{v^*,i}(x)) \right].$$

Denote by $x_{(-i)}$ the largest allocation in x after dropping x_i. The responder's expected payoff in a QRE with continuation value v^* is

$$\sum_{x \in X'} r^{\lambda}_{v^*,i}(x) \left[x_{(-i)} p^{\lambda}_{v^*,i}(x) + \delta v^* (1 - p^{\lambda}_{v^*,i}(x)) \right].$$

The difference between the proposer's and the responder's expected payoff captures the proposer's bargaining power. In the benchmark SSPE, the difference between the proposer's and the responder's expected payoff is

$$\left(1 - \frac{n-1}{2} \frac{\delta}{n}\right) - \frac{\delta}{n} = 1 - \frac{n+1}{2} \frac{\delta}{n}.$$

For $n = 3$ and $\delta \in \{\frac{1}{2}, 1\}$ this difference is, respectively, $\frac{2}{3}$ and $\frac{1}{3}$. For $n = 5$ and $\delta \in \{\frac{1}{2}, 1\}$ this difference is, respectively, $\frac{7}{10}$ and $\frac{2}{5}$. These benchmark values can be read on the figures below.

Since a non-proposing player is not certain to receive the largest allocation among the allocations the proposer distributes among the remaining players, denote the non-proposer's expected payoff in a QRE with continuation value v^* by

$$\sum_{x \in X'} r^{\lambda}_{v^*,i}(x) \left[x_j p^{\lambda}_{v^*,i}(x) + \delta v^* (1 - p^{\lambda}_{v^*,i}(x)) \right]$$

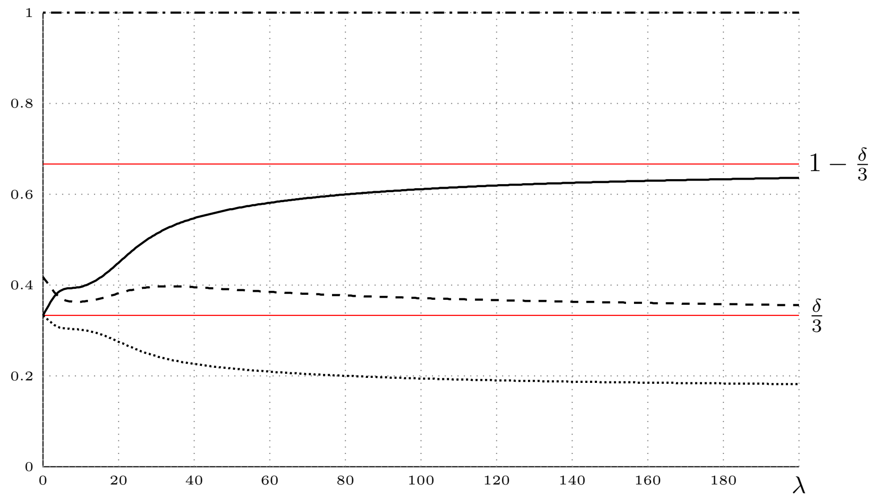
where $j \neq i$. In the benchmark SSPE, the difference between the proposer's and the non-proposer's expected payoff is

$$\left(1 - \frac{n-1}{2} \frac{\delta}{n}\right) - \frac{\delta}{n} \frac{1}{2} = 1 - \frac{\delta}{2}.$$

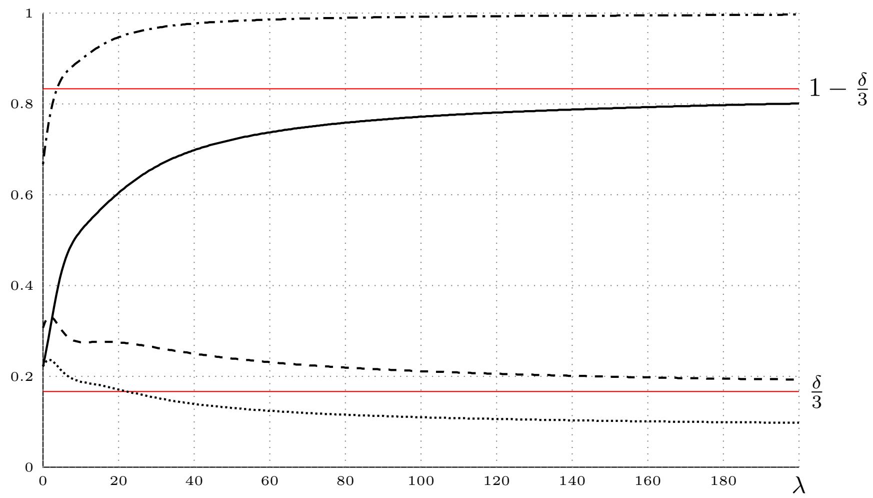
For $\delta \in \{\frac{1}{2}, 1\}$ the difference is, respectively, $\frac{1}{2}$ and $\frac{3}{4}$.

Figure 18: QRE in Baron and Ferejohn (1989) . . . Expected proposer's and responder's payoff

(a) $n = 3$ & $\delta = 1$



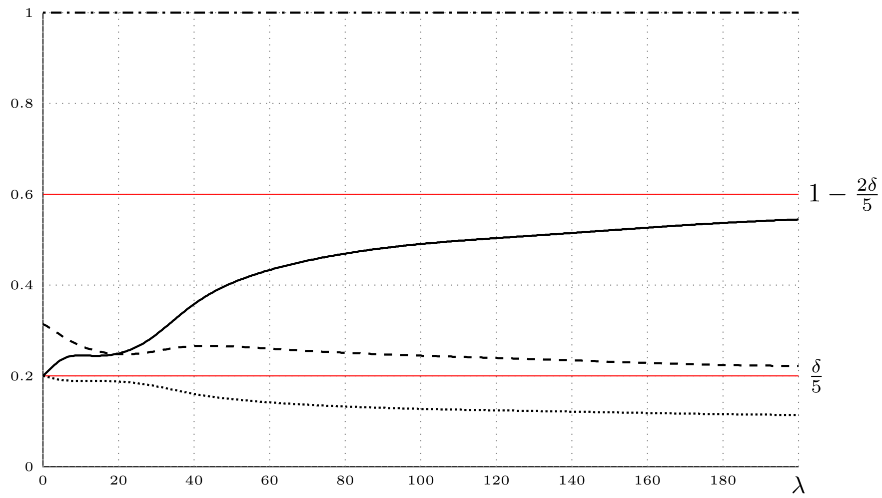
(b) $n = 3$ & $\delta = \frac{1}{2}$



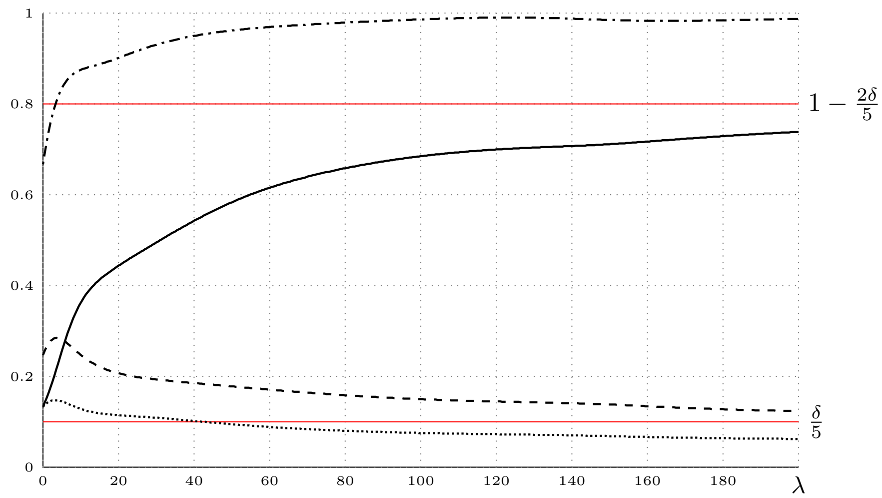
Note: Proposer's (solid line), responder's (dashed line) and non-proposer's (dotted line) expected payoff in QRE. The dash-dotted line sums one proposer's and two non-proposer's expected payoffs. The horizontal lines are the proposer's and responder's share in the benchmark SSPE.

Figure 19: QRE in Baron and Ferejohn (1989) . . . Expected proposer's and responder's payoff

(a) $n = 5$ & $\delta = 1$



(b) $n = 5$ & $\delta = \frac{1}{2}$



Note: Proposer's (solid line), responder's (dashed line) and non-proposer's (dotted line) expected payoff in QRE. The dash-dotted line sums one proposer's and four non-proposer's expected payoffs. The horizontal lines are the proposer's and responder's share in the benchmark SSPE.

6 QRE & QGF Estimation with (In)Experienced Subjects, All Rounds and Paths of Play

This section presents the MLE estimates of $\hat{\lambda}$ for the QRE model and $\{\hat{\lambda}, \hat{\phi}\}$ for the QGF model using alternative datasets relative to the results included in the paper. Table 1 reports the benchmark estimates presented here for comparison. To reiterate, these estimates use all round 1 proposals.

Table 2 reports estimates $\hat{\lambda}$ and $\{\hat{\lambda}, \hat{\phi}\}$ obtained using all round 1 proposals from experiments with inexperienced subjects (dropping periods 11-15 in experiment 1, 6-10 in experiment 6 and all data with re-invited subjects in experiments 2, 3, 4 and 5). Table 3 presents similar estimates but using all round 1 proposals from experiments with experienced subject.

Table 4 shows estimates $\hat{\lambda}$ and $\{\hat{\lambda}, \hat{\phi}\}$ obtained using all proposals from all rounds. The inclusion of data from different rounds requires a slight alternation of the estimation strategy because a) QGF makes different predictions for odd and even rounds and b) both QRE and QGF predict different probability of proceeding beyond round 1 for different values of λ and ϕ.

We deal with a) in a straightforward manner. Each observation of proposing or voting behavior comes with information about the round in which it was and we use round specific QGF predictions to calculate the maximum likelihood. For the variables in Table 4 that in the QGF model depend on odd and even rounds (average proposer's share, quartiles, share of minimum winning coalitions) we present odd and even round specific predictions, weighted by the probability the game ends in odd and even rounds.

Dealing with b) is slightly more complicated. For a given observation of proposing or voting behavior in round $r > 1$, we multiply the probability of that action being taken, as predicted by QRE or QGF, by the probability of the game proceeding to round r, also calculated using the QRE or QGF prediction. In other words, if the dataset contains observations from rounds $r > 1$, the MLE estimation penalizes high values of λ, which predict low rejection probabilities.

Finally, Table 5 presents the estimates using the observed paths of play, i.e., selected proposals only. In each experiment and each period, we observe a series of proposed allocations $x^1, \ldots, x^T$ containing $T - 1$ rejected proposals $x^1, \ldots, x^{T-1}$ and one accepted proposal x^T. We call $x^1, \ldots, x^T$ the path of play. For any λ in the QRE model or $\{\lambda, \phi\}$ in the QGF model we can calculate the likelihood of a given path of play as the probability of the event that all $x^1, \ldots, x^{T-1}$ are proposed and rejected and x^T is proposed and accepted (using again odd and even round specific predictions in QGF). Summing over all the observed paths we obtain the likelihood of a given dataset, which we maximize in order to estimate $\hat{\lambda}$ or $\{\hat{\lambda}, \hat{\phi}\}$.

Table 1: Estimation results, round 1 data (benchmark included in the paper)

Experiment	1	2	3	4	5	6
N	5	3	3	3	5	3
δ	4/5	1	1	1/2	1	1
Observations	275	330	411	420	450	480
$X^{\star}_{PR}$	.680	.666	.666	.833	.600	.666
	Data					
$Avg(X_{PR})$	.338	.553	.522	.537	.409	.486
$Q1(X_{PR})$	.250	.500	.500	.500	.350	.420
$Q2(X_{PR})$	.350	.530	.500	.530	.400	.500
$Q3(X_{PR})$	.400	.600	.570	.600	.500	.570
% MWC	.422	.727	.793	.695	.853	.573
$\mathbb{P}$[delay]	.036	.318	.219	.093	.422	.269
	QRE					
$\widehat{\lambda}$	20.2	22.1	23.4	10.6	33.5	18.4
$Avg(\widehat{X}_{PR})$	.441	.527	.532	.627	.400	.512
$Q1(\widehat{X}_{PR})$	.350	.490	.500	.540	.350	.460
$Q2(\widehat{X}_{PR})$	.450	.550	.550	.640	.400	.540
$Q3(\widehat{X}_{PR})$	.500	.600	.600	.730	.450	.600
% MWC	.547	.605	.635	.424	.773	.516
$\mathbb{P}$[delay]	.324	.345	.313	.179	.353	.443
Ln(L) – Overall	-2434.1	-2380.4	-2903.3	-3432.5	-3012.7	-3849.9
Ln(L) – Proposing	-2369.3	-2300.1	-2800.7	-3346.2	-2920.9	-3662.4
Ln(L) – Voting	-64.8	-80.2	-102.7	-86.3	-91.8	-187.6
	QGF					
$\widehat{\lambda}$	21.7	21.7	22.5	13.3	33.8	18.8
$\widehat{\phi}$	1	3	1	1	1	1
$Avg(\widehat{X}_{PR})$	.421	.518	.497	.594	.387	.491
$Q1(\widehat{X}_{PR})$	.350	.480	.460	.530	.350	.440
$Q2(\widehat{X}_{PR})$	.450	.540	.510	.610	.400	.510
$Q3(\widehat{X}_{PR})$	.500	.590	.560	.680	.450	.570
% MWC	.553	.601	.637	.449	.794	.541
$\mathbb{P}$[delay]	.301	.339	.273	.152	.315	.382
Ln(L) – Overall	-2374.5	-2377.7	-2833.1	-3309.2	-2945.4	-3766.0
Ln(L) – Proposing	-2308.8	-2296.4	-2722.4	-3237.2	-2857.5	-3560.9
Ln(L) – Voting	-65.7	-81.3	-110.6	-72.1	-87.9	-205.2
	Likelihood Ratio Test (P-Values)					
Overall	0.0000	0.0213	0.0000	0.0000	0.0000	0.0000
Proposing	0.0000	0.0066	0.0000	0.0000	0.0000	0.0000
Voting	1.0000	1.0000	1.0000	0.0000	0.0053	1.0000

Note: Experiment 1 from Frechette, Kagel, and Lehrer (2003); Experiments 2 through 4 from Frechette, Kagel, and Morelli (2005b); Experiment 5 from Frechette, Kagel, and Morelli (2005a); Experiment 6 from Drouvelis, Montero, and Sefton (2010). X_{PR}, $X^{\star}_{PR}$, and $\widehat{X}_{PR}$ refer, respectively to the proposer's allocation observed in the data, the proposer's allocation predicted by the benchmark model, and the proposer's allocation predicted by the MLE estimates. % MWC refers to the incidence of minimum winning coalitions, defined as proposals where at least 1 member (for $n = 3$), or at least 2 members (for $n = 5$), receive less than 5% of the pie. All data and estimates refer to round 1 behavior.

Table 2: Estimation results, round 1 data, inexperienced subject only

Experiment	1	2	3	4	5	6
N	5	3	3	3	5	3
δ	4/5	1	1	1/2	1	1
Observations	200	240	249	300	300	240
$X^{\star}_{PR}$	.680	.666	.666	.833	.600	.666
	Data					
$Avg(X_{PR})$	.328	.552	.520	.517	.405	.465
$Q1(X_{PR})$	.250	.500	.500	.470	.350	.330
$Q2(X_{PR})$	.300	.530	.500	.500	.400	.500
$Q3(X_{PR})$	.400	.600	.600	.570	.500	.500
% MWC	.335	.700	.719	.630	.807	.417
$\mathbb{P}$[delay]	.050	.350	.265	.110	.383	.188
	QRE					
$\widehat{\lambda}$	18.9	20.9	21.7	9.0	32.3	15.7
$Avg(\widehat{X}_{PR})$	.438	.522	.525	.617	.394	.503
$Q1(\widehat{X}_{PR})$	.350	.480	.490	.520	.350	.430
$Q2(\widehat{X}_{PR})$	.450	.550	.550	.640	.400	.530
$Q3(\widehat{X}_{PR})$	.500	.600	.600	.740	.450	.610
% MWC	.539	.576	.595	.410	.747	.457
$\mathbb{P}$[delay]	.338	.377	.356	.196	.382	.508
Ln(L) – Overall	-1810.6	-1782.2	-1841.2	-2515.6	-2097.2	-2018.4
Ln(L) – Proposing	-1758.3	-1717.7	-1763.2	-2441.6	-2041.0	-1918.9
Ln(L) – Voting	-52.3	-64.5	-78.0	-74.0	-56.2	-99.5
	QGF					
$\widehat{\lambda}$	20.6	20.6	20.3	11.7	32.4	16.1
$\widehat{\phi}$	1	4	1	1	1	1
$Avg(\widehat{X}_{PR})$	.419	.516	.493	.591	.381	.486
$Q1(\widehat{X}_{PR})$	.350	.470	.450	.510	.350	.420
$Q2(\widehat{X}_{PR})$	.450	.540	.510	.600	.400	.510
$Q3(\widehat{X}_{PR})$	.500	.590	.560	.680	.450	.580
% MWC	.537	.573	.581	.419	.762	.473
$\mathbb{P}$[delay]	.315	.374	.336	.171	.347	.464
Ln(L) – Overall	-1770.0	-1780.8	-1818.0	-2443.2	-2060.3	-1992.7
Ln(L) – Proposing	-1717.7	-1715.8	-1736.8	-2380.7	-2006.0	-1882.4
Ln(L) – Voting	-52.2	-65.0	-81.2	-62.4	-54.3	-110.3
	Likelihood Ratio Test (P-Values)					
Overall	0.0000	0.0905	0.0000	0.0000	0.0000	0.0000
Proposing	0.0000	0.0458	0.0000	0.0000	0.0000	0.0000
Voting	0.7882	1.0000	1.0000	0.0000	0.0532	1.0000

Note: Experiment 1 from Frechette, Kagel, and Lehrer (2003); Experiments 2 through 4 from Frechette, Kagel, and Morelli (2005b); Experiment 5 from Frechette, Kagel, and Morelli (2005a); Experiment 6 from Drouvelis, Montero, and Sefton (2010). X_{PR}, $X^{\star}_{PR}$, and $\widehat{X}_{PR}$ refer, respectively to the proposer's allocation observed in the data, the proposer's allocation predicted by the benchmark model, and the proposer's allocation predicted by the MLE estimates. % MWC refers to the incidence of minimum winning coalitions, defined as proposals where at least 1 member (for $n = 3$), or at least 2 members (for $n = 5$), receive less than 5% of the pie. All data and estimates refer to round 1 behavior.

Table 3: Estimation results, round 1 data, experienced subject only

Experiment	1	2	3	4	5	6
N	5	3	3	3	5	3
δ	4/5	1	1	1/2	1	1
Observations	75	90	162	120	150	240
$X^{\star}_{PR}$	.680	.666	.666	.833	.600	.666
	Data					
$Avg(X_{PR})$	.366	.556	.525	.588	.419	.507
$Q1(X_{PR})$	.350	.530	.500	.530	.350	.460
$Q2(X_{PR})$	.400	.570	.530	.580	.400	.500
$Q3(X_{PR})$	.400	.570	.570	.670	.450	.580
% MWC	.653	.800	.907	.858	.947	.729
$\mathbb{P}$[delay]	.000	.233	.148	.050	.500	.350
	QRE					
$\widehat{\lambda}$	23.4	25.6	26.3	15.3	36.0	21.2
$Avg(\widehat{X}_{PR})$	.447	.540	.543	.643	.414	.523
$Q1(\widehat{X}_{PR})$	.400	.510	.510	.580	.400	.480
$Q2(\widehat{X}_{PR})$	.450	.560	.560	.650	.450	.550
$Q3(\widehat{X}_{PR})$	.500	.600	.600	.710	.450	.600
% MWC	.587	.684	.698	.494	.823	.583
$\mathbb{P}$[delay]	.287	.263	.249	.134	.298	.369
Ln(L) – Overall	-617.9	-591.4	-1052.3	-891.0	-907.7	-1812.4
Ln(L) – Proposing	-604.9	-576.0	-1028.8	-878.5	-871.7	-1722.0
Ln(L) – Voting	-13.0	-15.4	-23.5	-12.5	-36.0	-90.4
	QGF					
$\widehat{\lambda}$	24.4	25.1	28.1	18.4	37.0	21.8
$\widehat{\phi}$	1	3	1	1	1	1
$Avg(\widehat{X}_{PR})$	.427	.530	.505	.611	.400	.496
$Q1(\widehat{X}_{PR})$	.350	.500	.470	.560	.350	.450
$Q2(\widehat{X}_{PR})$	.450	.540	.510	.620	.400	.510
$Q3(\widehat{X}_{PR})$	.500	.580	.550	.670	.450	.560
% MWC	.604	.680	.753	.575	.855	.620
$\mathbb{P}$[delay]	.267	.257	.166	.106	.255	.292
Ln(L) – Overall	-599.8	-589.7	-983.8	-838.5	-873.9	-1747.6
Ln(L) – Proposing	-586.0	-574.2	-955.6	-828.8	-840.1	-1654.0
Ln(L) – Voting	-13.8	-15.5	-28.2	-9.7	-33.8	-93.6
	Likelihood Ratio Test (P-Values)					
Overall	0.0000	0.0619	0.0000	0.0000	0.0000	0.0000
Proposing	0.0000	0.0580	0.0000	0.0000	0.0000	0.0000
Voting	1.0000	1.0000	1.0000	0.0178	0.0359	1.0000

Note: Experiment 1 from Frechette, Kagel, and Lehrer (2003); Experiments 2 through 4 from Frechette, Kagel, and Morelli (2005b); Experiment 5 from Frechette, Kagel, and Morelli (2005a); Experiment 6 from Drouvelis, Montero, and Sefton (2010). X_{PR}, $X^{\star}_{PR}$, and $\widehat{X}_{PR}$ refer, respectively to the proposer's allocation observed in the data, the proposer's allocation predicted by the benchmark model, and the proposer's allocation predicted by the MLE estimates. % MWC refers to the incidence of minimum winning coalitions, defined as proposals where at least 1 member (for $n = 3$), or at least 2 members (for $n = 5$), receive less than 5% of the pie. All data and estimates refer to round 1 behavior.

Table 4: Estimation results, all data

Experiment	1	2	3	4	5	6
N	5	3	3	3	5	3
δ	4/5	1	1	1/2	1	1
Observations	285	486	555	474	745	678
$X^{\star}_{PR}$	.680	.666	.666	.833	.600	.666
	Data					
$Avg(X_{PR})$	.333	.556	.520	.538	.405	.491
$Q1(X_{PR})$	.250	.500	.500	.500	.350	.420
$Q2(X_{PR})$	.350	.530	.500	.530	.400	.500
$Q3(X_{PR})$	.400	.600	.570	.600	.500	.580
% MWC	.407	.739	.791	.679	.840	.603
$\mathbb{P}$[delay]	.036	.318	.219	.093	.422	.269
	QRE					
$\widehat{\lambda}$	19.4	18.1	19.8	9.7	29.2	16.1
$Avg(\widehat{X}_{PR})$	.439	.511	.517	.622	.376	.504
$Q1(\widehat{X}_{PR})$	.350	.460	.470	.530	.350	.430
$Q2(\widehat{X}_{PR})$	.450	.540	.540	.640	.400	.530
$Q3(\widehat{X}_{PR})$	.500	.600	.600	.730	.450	.610
% MWC	.541	.509	.550	.416	.680	.465
$\mathbb{P}$[delay]	.333	.451	.406	.189	.459	.499
Ln(L) – Overall	-2567.7	-3828.9	-4231.7	-4063.1	-5582.7	-5715.2
Ln(L) – Proposing	-2488.8	-3626.7	-4015.3	-3897.8	-5286.3	-5372.6
Ln(L) – Voting	-78.9	-202.2	-216.4	-165.2	-296.4	-342.6
	QGF					
$\widehat{\lambda}$	20.9	18.0	19.2	11.9	29.1	16.1
$\widehat{\phi}$	1	6	1	1	1	1
$Avg(\widehat{X}_{PR})$	.427	.509	.500	.599	.369	.495
$Q1(\widehat{X}_{PR})$	.350	.450	.450	.520	.300	.420
$Q2(\widehat{X}_{PR})$	.450	.540	.520	.610	.400	.520
$Q3(\widehat{X}_{PR})$	.500	.600	.580	.690	.450	.600
% MWC	.545	.508	.546	.425	.682	.469
$\mathbb{P}$[delay]	.312	.448	.370	.168	.434	.464
Ln(L) – Overall	-2511.5	-3828.4	-4181.5	-3957.0	-5541.0	-5663.5
Ln(L) – Proposing	-2431.6	-3625.2	-3950.9	-3801.6	-5237.6	-5299.0
Ln(L) – Voting	-79.9	-203.3	-230.5	-155.4	-303.3	-364.5
	Likelihood Ratio Test (P-Values)					
Overall	0.0000	0.3390	0.0000	0.0000	0.0000	0.0000
Proposing	0.0000	0.0808	0.0000	0.0000	0.0000	0.0000
Voting	1.0000	1.0000	1.0000	0.0000	1.0000	1.0000

Note: Experiment 1 from Frechette, Kagel, and Lehrer (2003); Experiments 2 through 4 from Frechette, Kagel, and Morelli (2005b); Experiment 5 from Frechette, Kagel, and Morelli (2005a); Experiment 6 from Drouvelis, Montero, and Sefton (2010). X_{PR}, $X^{\star}_{PR}$, and $\widehat{X}_{PR}$ refer, respectively to the proposer's allocation observed in the data, the proposer's allocation predicted by the benchmark model, and the proposer's allocation predicted by the MLE estimates. % MWC refers to the incidence of minimum winning coalitions, defined as proposals where at least 1 member (for $n = 3$), or at least 2 members (for $n = 5$), receive less than 5% of the pie. All data and estimates refer to all rounds of behavior.

Table 5: Estimation results, paths of play

Experiment	1	2	3	4	5	6
N	5	3	3	3	5	3
δ	4/5	1	1	1/2	1	1
Observations	57/55	162/110	185/137	158/140	149/90	226/160
$X^{\star}_{PR}$	.680	.666	.666	.833	.600	.666
	Data					
$Avg(X_{PR})$	.323	.556	.525	.552	.416	.495
$Q1(X_{PR})$	.200	.500	.500	.500	.350	.420
$Q2(X_{PR})$	.350	.530	.500	.530	.400	.500
$Q3(X_{PR})$	.400	.600	.570	.600	.500	.580
% MWC	.421	.716	.805	.709	.819	.655
$\mathbb{P}$[delay]	.036	.318	.219	.093	.422	.269
	QRE					
$\widehat{\lambda}$	20.0	20.5	23.5	10.7	32.5	19.7
$Avg(\widehat{X}_{PR})$	.440	.520	.533	.627	.395	.517
$Q1(\widehat{X}_{PR})$	.350	.480	.500	.540	.350	.470
$Q2(\widehat{X}_{PR})$	.450	.540	.550	.640	.400	.540
$Q3(\widehat{X}_{PR})$	.500	.600	.600	.730	.450	.600
% MWC	.545	.567	.638	.425	.751	.547
$\mathbb{P}$[delay]	.326	.387	.310	.178	.377	.409
Ln(L) – Overall	-507.4	-1248.5	-1334.5	-1285.3	-1060.2	-1784.9
	QGF					
$\widehat{\lambda}$	21.7	19.5	23.3	13.1	32.5	20.4
$\widehat{\phi}$	1	1	1	1	1	1
$Avg(\widehat{X}_{PR})$	.428	.501	.507	.600	.386	.502
$Q1(\widehat{X}_{PR})$	.350	.450	.470	.530	.350	.450
$Q2(\widehat{X}_{PR})$	.450	.520	.520	.610	.400	.520
$Q3(\widehat{X}_{PR})$	.500	.580	.570	.680	.450	.580
% MWC	.555	.554	.650	.446	.760	.577
$\mathbb{P}$[delay]	.301	.360	.253	.154	.345	.333
Ln(L) – Overall	-493.2	-1240.9	-1296.8	-1243.7	-1047.5	-1739.3
	Likelihood Ratio Test (P-Values)					
Overall	0.0000	0.0001	0.0000	0.0000	0.0000	0.0000

Note: Experiment 1 from Frechette, Kagel, and Lehrer (2003); Experiments 2 through 4 from Frechette, Kagel, and Morelli (2005b); Experiment 5 from Frechette, Kagel, and Morelli (2005a); Experiment 6 from Drouvelis, Montero, and Sefton (2010). X_{PR}, $X^{\star}_{PR}$, and $\widehat{X}_{PR}$ refer, respectively to the proposer's allocation observed in the data, the proposer's allocation predicted by the benchmark model, and the proposer's allocation predicted by the MLE estimates. % MWC refers to the incidence of minimum winning coalitions, defined as proposals where at least 1 member (for $n = 3$), or at least 2 members (for $n = 5$), receive less than 5% of the pie. All data and estimates refer to observed paths of play, i.e., selected proposals only. Number of observations overall/number of paths.

7 Log-likelihood of SSPE benchmark

The table below displays log-likelihood and summary statistics for two models. The first model is QRE at the benchmark $\widehat{\lambda}$ estimates. The second model is QRE at $\widehat{\lambda} = 500$, a close approximation of the benchmark SSPE. The bottom panel of the table shows results of a likelihood ratio test of a null that both models fit the experimental data equally well.

Table 6: Likelihood ratio test: QRE at benchmark $\widehat{\lambda}$ vs. SSPE (QRE at $\widehat{\lambda} = 500$)

Experiment	1	2	3	4	5	6
N	5	3	3	3	5	3
δ	4/5	1	1	1/2	1	1
Observations	275	330	411	420	450	480
$X^{\star}_{PR}$	.680	.666	.666	.833	.600	.666
			Data			
$Avg(X_{PR})$	.338	.553	.522	.537	.409	.486
% MWC	.422	.727	.793	.695	.853	.573
$\mathbb{P}$[delay]	.036	.318	.219	.093	.422	.269
			QRE			
$\widehat{\lambda}$	20.2	22.1	23.4	10.6	33.5	18.4
$Avg(\widehat{X}_{PR})$	.441	.527	.532	.627	.400	.512
% MWC	.547	.605	.635	.424	.773	.516
$\mathbb{P}$[delay]	.324	.345	.313	.179	.353	.443
Ln(L) – Overall	-2434.1	-2380.4	-2903.3	-3432.5	-3012.7	-3849.9
Ln(L) – Proposing	-2369.3	-2300.1	-2800.7	-3346.2	-2920.9	-3662.4
Ln(L) – Voting	-64.8	-80.2	-102.7	-86.3	-91.8	-187.6
			SSPE (large $\widehat{\lambda}$)			
$\widehat{\lambda}$	500	500	500	500	500	500
$Avg(\widehat{X}_{PR})$	.600	.653	.653	.820	.500	.653
% MWC	1.00	1.00	1.00	1.00	1.00	1.00
$\mathbb{P}$[delay]	.000	.012	.012	.001	.000	.012
Ln(L) – Overall	-40715.2	-30962.3	-35872.4	-65342.3	-32182.9	-51178.1
Ln(L) – Proposing	-40443.9	-30422.0	-35139.8	-64611.7	-31350.0	-49805.8
Ln(L) – Voting	-271.3	-540.3	-732.5	-730.6	-832.9	-1372.3
			Likelihood Ratio Test (P-Values)			
Overall	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Proposing	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
Voting	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

Note: Experiment 1 from Frechette, Kagel, and Lehrer (2003); Experiments 2 through 4 from Frechette, Kagel, and Morelli (2005b); Experiment 5 from Frechette, Kagel, and Morelli (2005a); Experiment 6 from Drouvelis, Montero, and Sefton (2010). X_{PR}, $X^{\star}_{PR}$, and $\widehat{X}_{PR}$ refer, respectively to the proposer's allocation observed in the data, the proposer's allocation predicted by the benchmark model, and the proposer's allocation predicted by the MLE estimates. % MWC refers to the incidence of minimum winning coalitions, defined as proposals where at least 1 member (for $n = 3$), or at least 2 members (for $n = 5$), receive less than 5% of the pie. All data and estimates refer to round 1 behavior.

8 Proposals Distribution and Acceptance Probabilities in QRE

The first three figures below show the *pdf* of proposals made by player 3 in the QRE with $n = 3$ and $\delta = 1$ for $\lambda \in \{2, 10, 32\}$. The second three figures show the probability of acceptance for the same parameters. The location of the vertical lines in the figures corresponds to the SSPE in the benchmark model.

Figure 20: QRE proposal *pdf*
$N = 3$, $\delta = 1$, $\lambda = 2$

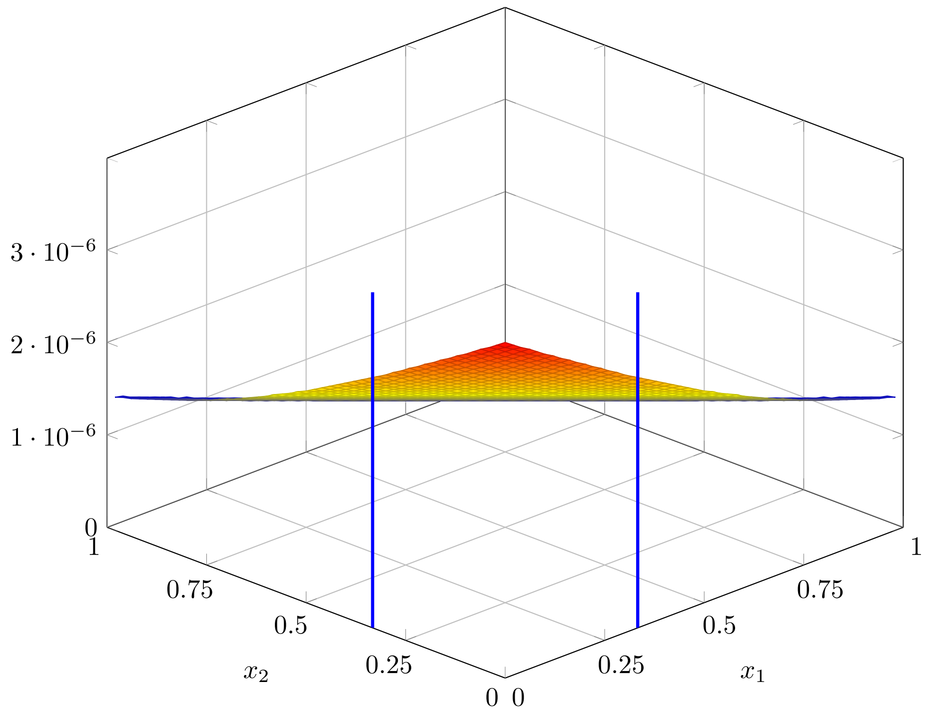


Figure 21: QRE proposal *pdf*
$N = 3$, $\delta = 1$, $\lambda = 10$

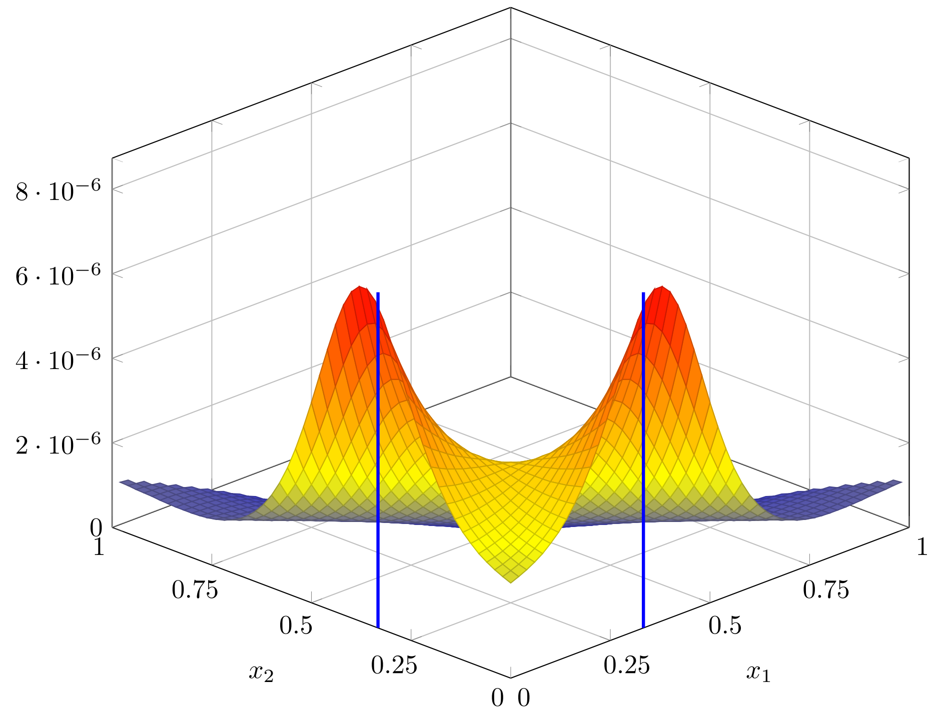


Figure 22: QRE proposal *pdf*
$N = 3$, $\delta = 1$, $\lambda = 32$

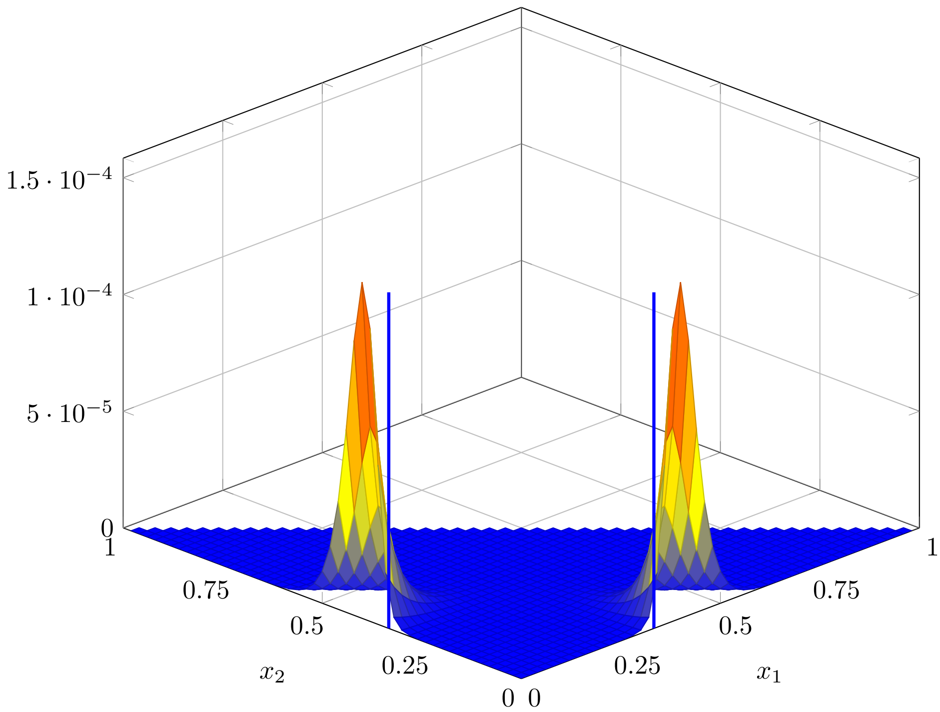


Figure 23: QRE probability of acceptance
$N = 3$, $\delta = 1$, $\lambda = 2$

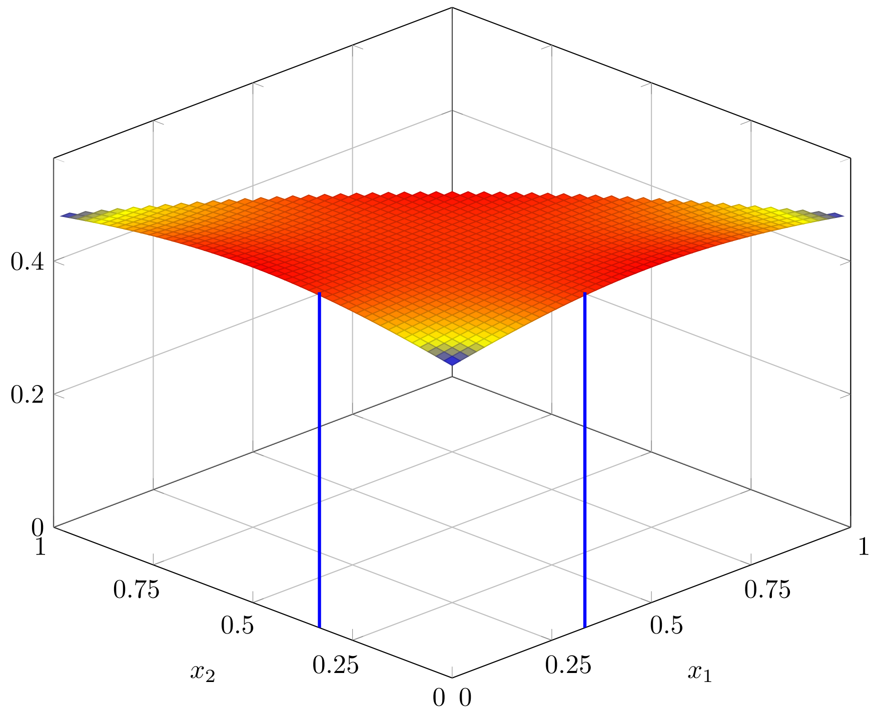


Figure 24: QRE probability of acceptance
$N = 3$, $\delta = 1$, $\lambda = 10$

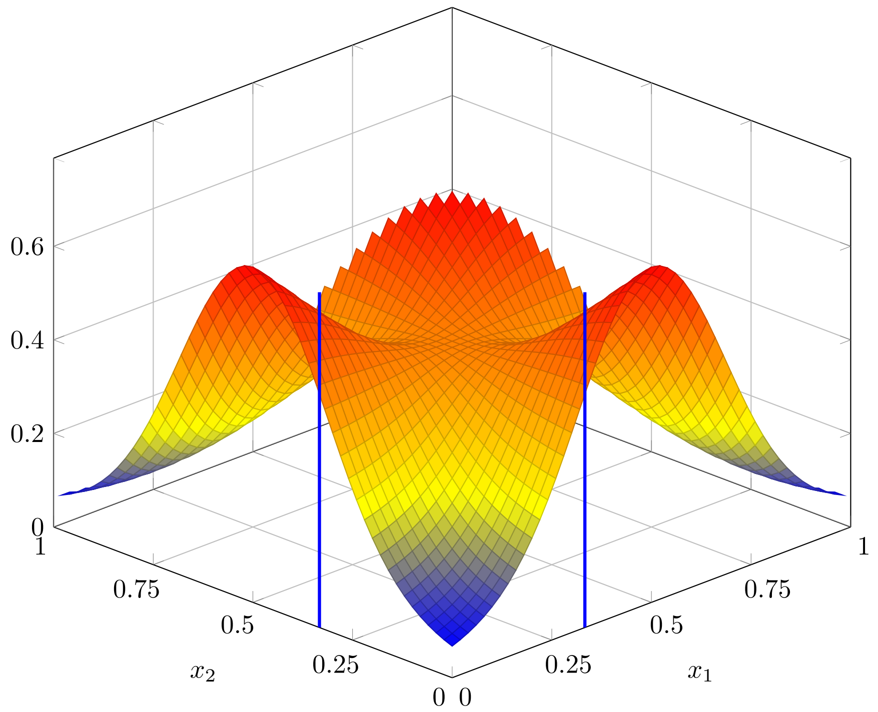
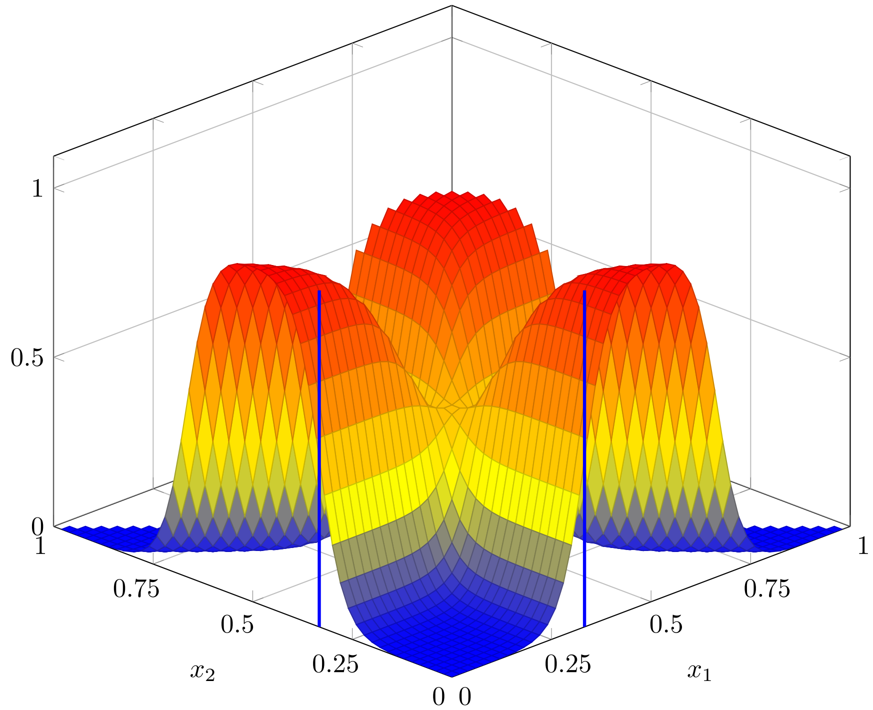


Figure 25: QRE probability of acceptance
$N = 3$, $\delta = 1$, $\lambda = 32$



9 Proposals Distribution in Experimental Data

The six figures below show the distribution of the experimental round 1 proposals. The data has been reorganized such that player 1 is the proposer, hence the figures show allocations to players 2 and 3. For experiments with $n = 5$ we summed $x_2 + x_3$ and $x_4 + x_5$ in order to be able to plot the data, where, for a given observation, x_2 and x_3 (x_4 and x_5) denote the two highest (lowest) allocations to the non-proposing players. The distribution has been rescaled to be symmetric around the $x_2 = x_3$ or $x_2 + x_3 = x_4 + x_5$ axis. The location of the vertical lines in the figures corresponds to the SSPE in the benchmark model.

Figure 26: Experiment 1, round 1 proposal distribution, proposing player 1

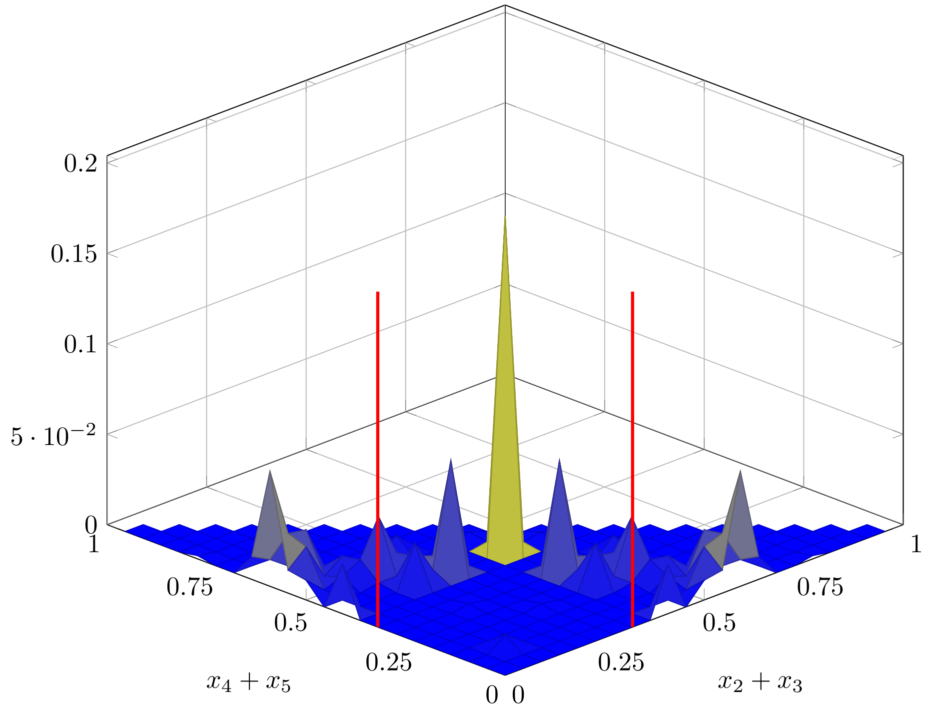


Figure 27: Experiment 2, round 1 proposal distribution, proposing player 1

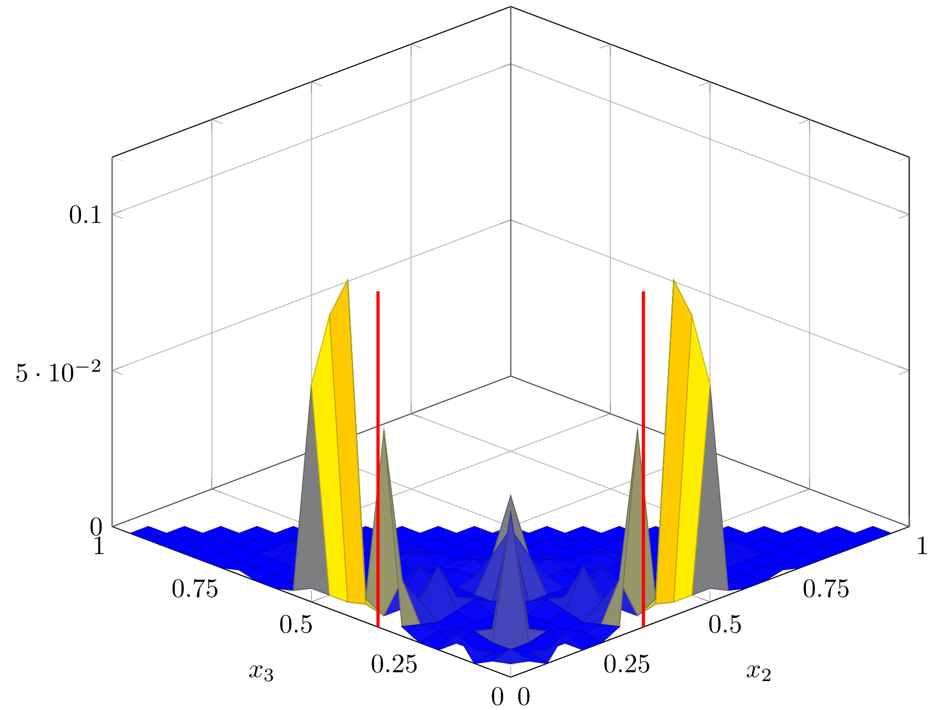


Figure 28: Experiment 3, round 1 proposal distribution, proposing player 1

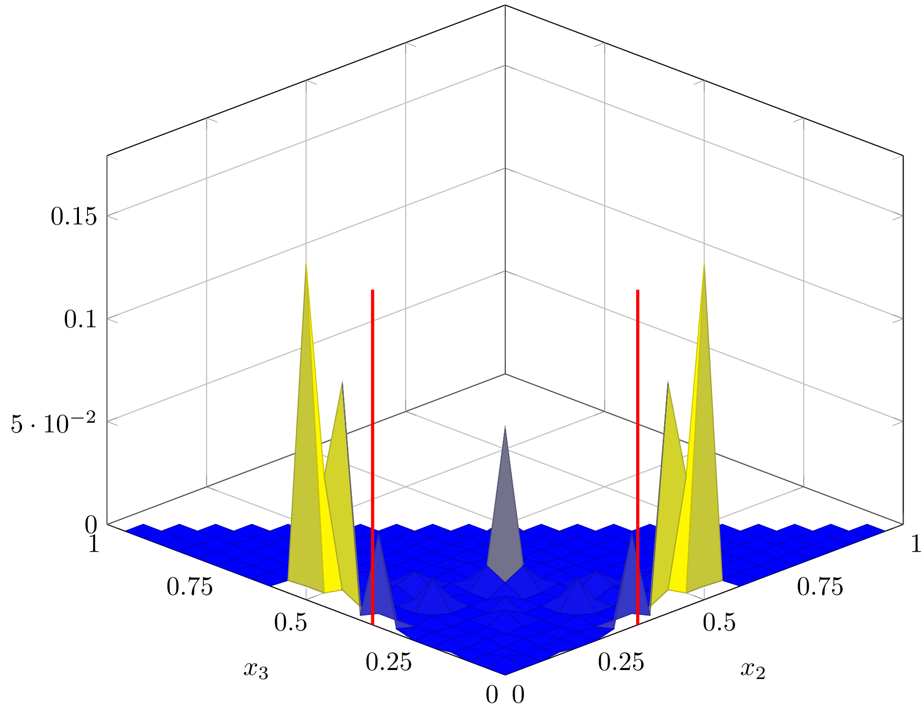


Figure 29: Experiment 4, round 1 proposal distribution, proposing player 1

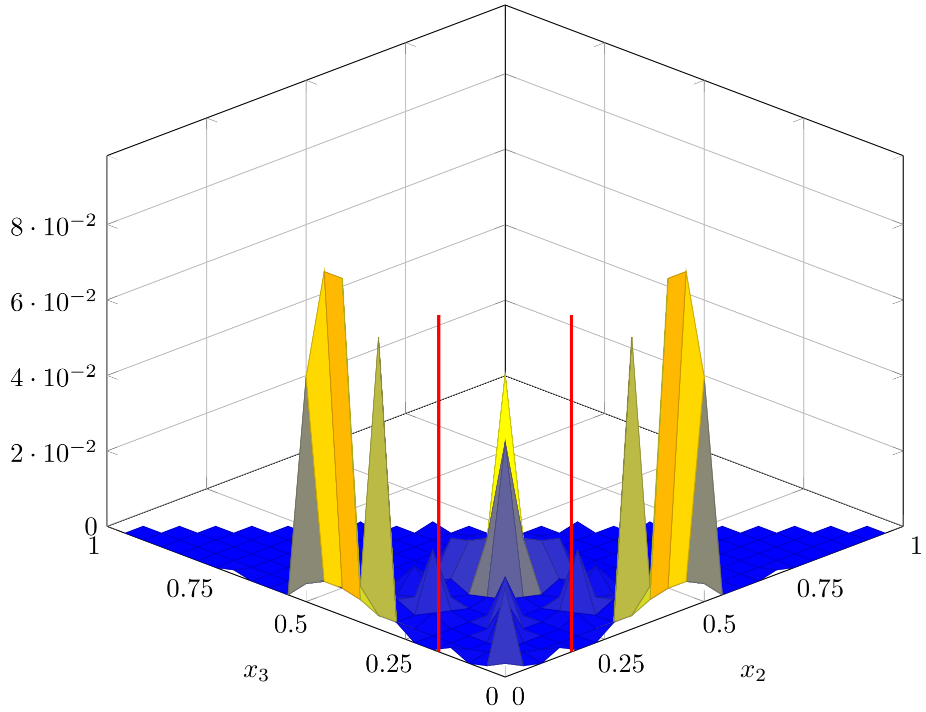


Figure 30: Experiment 5, round 1 proposal distribution, proposing player 1

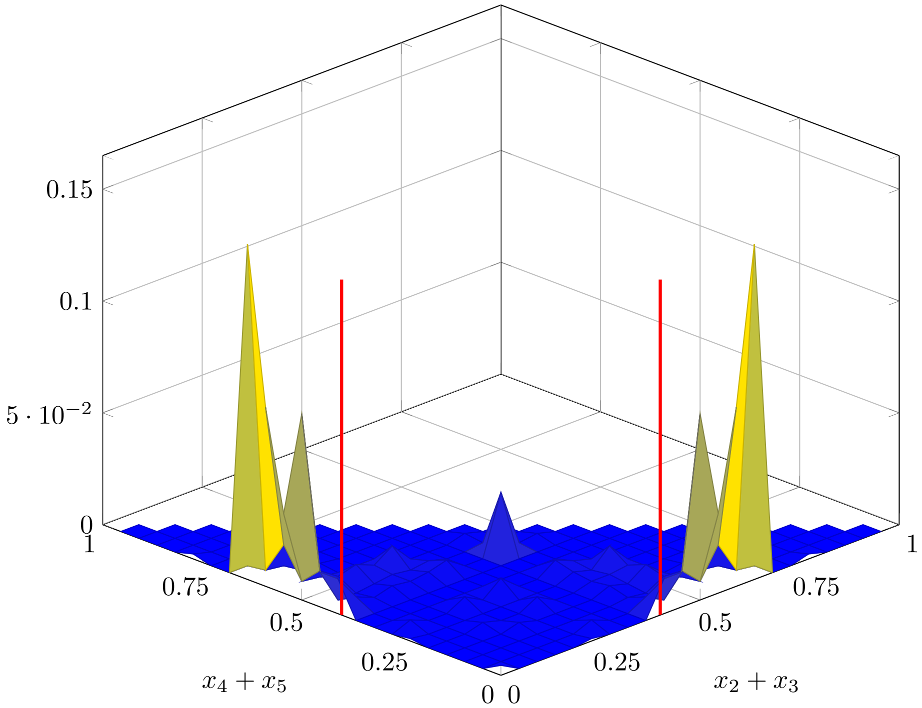
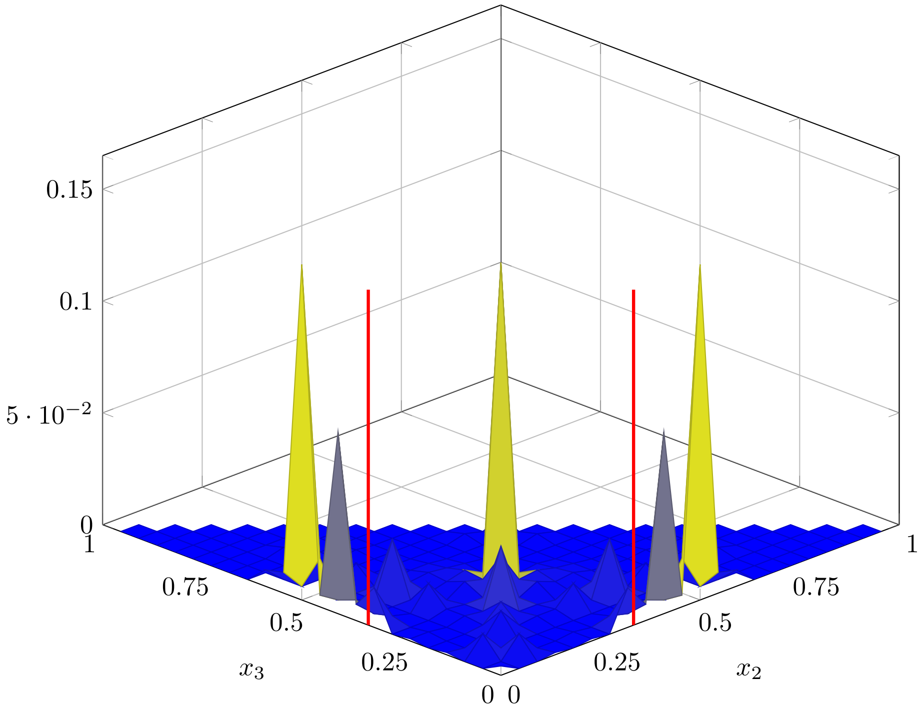


Figure 31: Experiment 6, round 1 proposal distribution, proposing player 1



10 Stationary Version of Gambler's Fallacy Model

We can model Gambler's Fallacy with an alternative assumption: with probability $\phi \in [0,1]$ the current proposer is excluded from the proposer recognitions in the following period and with probability $1-\phi$ the current proposer is included in the proposer recognitions in the following period. The rest of the model is as described in the paper.

The structure of the equilibrium clearly remains the same as in the benchmark model. The proposer allocates nothing to $\frac{n-1}{2}$ players, allocates a share that ensures a positive vote to $\frac{n-1}{2}$ players and keeps what remains for herself. Denote by x the equilibrium share a responder receives from the proposer if she is a member of the coalition of players supporting the proposal. Given the equilibrium continuation value of the respondents, v_r, we have $x = \delta v_r$.

A player's continuation value conditional on being recognized to propose, v_p, is:

$$\begin{aligned} v_p &= \phi\left[\frac{1}{2}x\right] + (1-\phi)\left[\frac{1}{n}\left(1-\frac{n-1}{2}x\right) + \frac{n-1}{n}\frac{1}{2}x\right] \\ &= \frac{2+\phi(xn-2)}{2n} \end{aligned} \tag{1}$$

A player's continuation value conditional on not being recognized to propose, v_r, is

$$\begin{aligned} v_r &= \phi\left[\frac{1}{n-1}\left(1-\frac{n-1}{2}x\right) + \frac{n-2}{n-1}\frac{1}{2}x\right] \\ &\quad + (1-\phi)\left[\frac{1}{n}\left(1-\frac{n-1}{2}x\right) + \frac{n-1}{n}\frac{1}{2}x\right] \\ &= \frac{\phi(1-\frac{x}{2})}{n-1} + \frac{1-\phi}{n}. \end{aligned} \tag{2}$$

Solving $x = \delta v_r$ for x gives

$$x = \frac{\delta}{n}\frac{2n-2+2\phi}{2n-2+\delta\phi}$$

which equals $\frac{\delta}{n}$ when $\phi = 0$ (as in the benchmark model) and is clearly increasing in ϕ.

The last thing we need to check is that $1-\frac{n-1}{2}x > \delta v_p$, that is, the proposer attains a higher payoff by allocating x to $\frac{n-1}{2}$ players rather than proposing an allocation which would be rejected and then receiving, in expectation, δv_p. Substituting the expression for v_p from above into $1-\frac{n-1}{2}x > \delta v_p$ gives

$$x < \frac{2}{n}\frac{n+\delta\phi-\delta}{n+\delta\phi-1}$$

which combined with $x = \frac{2\delta}{n}\frac{n-1+\phi}{2n-2+\delta\phi}$ rewrites as

$$n(n(2-\delta)-2) + \delta\phi(n(2-\delta)-1) + \delta > 0$$

which clearly holds since $n(2-\delta) \geq 3$ for any $\delta \in [0,1]$ and $n \in \mathbb{N}_{\geq 3}$.

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